

# Superstring Scattering from D-Branes

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## Abstract

We derive fully covariant expressions for all two-point scattering amplitudes of two massless closed strings from a Dirichlet  $p$ -brane. This construction relies on the observation that there is a simple relation between these D-brane amplitudes in type II superstring theory and four-point scattering amplitudes for type I open superstrings. From the two-point amplitudes, we derive the long range background fields for the D-branes, and verify that as expected they correspond to those of extremally charged  $p$ -brane solutions of the low energy effective action.

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# 1 Introduction

Recent exciting progress in string theory has revealed many new connections between superstring theories which had previously been regarded as distinct theories[1]. In fact it may be that all string theories are different phases of a single underlying theory in eleven or twelve dimensions [2]. Within these discussions, extended objects, other than just strings, play an important role. Hence these developments have generated a renewed interest in  $p$ -branes (*i.e.*,  $p$ -dimensional extended objects) and their interactions.

In type II superstring theories, there is a remarkably simple description of  $p$ -branes carrying Ramond–Ramond (R-R) charges[3]. The string background is taken to be simply flat empty space, however interactions of closed superstrings with these  $p$ -branes are described by world-sheets with boundaries fixed to a particular surface at the position of a  $p$ -brane. The latter is accomplished by imposing Dirichlet boundary conditions on the world-sheet fields [4, 5]. Hence these objects are referred to as Dirichlet  $p$ -branes ( $Dp$ -branes) or generically as simply D-branes. Within the type IIA theory, the  $Dp$ -branes can have  $p = 0, 2, 4, 6$  or  $8$ , while for the type IIB strings,  $p$  ranges over  $-1, 1, 3, 5, 7, 9$  [3]. So far there have been only limited results in calculating the scattering amplitudes describing the interactions of closed strings with D-branes[6, 7, 8], and the present paper provides an extension of these previous works.

The paper is organized as follows: In the following section we describe the calculation of the scattering of two massless states in the Neveu-Schwarz–Neveu-Schwarz (NS-NS) closed string sector from a Dirichlet  $p$ -brane using conformal field theory techniques. Our result extends the calculation of Klebanov and Thorlacius [6] to fully covariant amplitudes (without any restrictions on the polarization tensor). We conclude this section by observing that the above calculation exactly parallels that of a four-point amplitude of massless NS states for the open superstring. This observation then provides a general method for the construction of any two-point D-brane amplitude. In section 3 we present all other two-point amplitudes for scattering from a Dirichlet  $p$ -brane by using previously calculated open superstring amplitudes. Our results include the scattering amplitudes with bosonic R-R states, and also fermionic NS-R and R-NS states. In section 4, we examine the massless closed string poles in these amplitudes. By comparing these terms to those in analogous field theory calculations, we are able to extract the long range background fields surrounding a  $Dp$ -branes. Our calculations verify that these fields do correspond to those of extremally charged  $p$ -brane solutions of the low energy theory. We conclude with a discussion of our results in section 5. Appendices A and B contain some useful information on the conventions used in our calculations.

## 2 NS-NS scattering amplitudes

We begin by calculating the amplitudes describing the scattering of two massless NS-NS states from a Dirichlet  $p$ -brane (*i.e.*, the scattering of gravitons, dilatons or Kalb-Ramond (antisymmetric tensor) states). The amplitudes are calculated as two closed string vertex

operator insertions on a disk with appropriate boundary conditions [4, 5]. For a D $p$ -brane,<sup>1</sup> standard Neumann boundary conditions are imposed at the disk boundary on the world-sheet fields associated with the  $p + 1$  directions parallel to the brane's world-volume. The fields associated with the remaining  $9 - p$  coordinates orthogonal to the brane satisfy Dirichlet boundary conditions, which fixes the world-sheet boundary to the  $p$ -brane. Using a conformal transformation, the amplitude can be represented as a calculation in the upper half of the complex plane with the real axis as the world-sheet boundary. The amplitude may then be written as

$$A \simeq \int d^2 z_1 d^2 z_2 \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2) \rangle \quad (1)$$

where the vertex operators are

$$\begin{aligned} V_1(z_1, \bar{z}_1) &= \varepsilon_{1\mu\nu} :V_{-1}^\mu(p_1, z_1): :\tilde{V}_{-1}^\nu(p_1, \bar{z}_1): \\ V_2(z_2, \bar{z}_2) &= \varepsilon_{2\mu\nu} :V_0^\mu(p_2, z_2): :\tilde{V}_0^\nu(p_2, \bar{z}_2): . \end{aligned} \quad (2)$$

The holomorphic components above are given by

$$\begin{aligned} V_{-1}^\mu(p_1, z_1) &= e^{-\phi(z_1)} \psi^\mu(z_1) e^{ip_1 \cdot X(z_1)} \\ V_0^\mu(p_2, z_2) &= (\partial X^\mu(z_2) + ip_2 \cdot \psi(z_2) \psi^\mu(z_2)) e^{ip_2 \cdot X(z_2)} . \end{aligned} \quad (3)$$

The antiholomorphic components take the same form as in eq. (3) but with the left-moving fields replaced by their right-moving counterparts – *i.e.*,  $X(z) \rightarrow \tilde{X}(\bar{z})$ ,  $\psi(z) \rightarrow \tilde{\psi}(\bar{z})$ , and  $\phi(z) \rightarrow \tilde{\phi}(\bar{z})$ . As usual, the momenta and polarization tensors satisfy

$$p_i^2 = 0 , \quad p_i^\mu \varepsilon_{i\mu\nu} = 0 = \varepsilon_{i\mu\nu} p_i^\nu .$$

and the various physical states would be represented with

$$\begin{aligned} \text{graviton :} & \quad \varepsilon_{i\mu\nu} = \varepsilon_{i\nu\mu}, \quad \varepsilon_{i\mu}{}^\mu = 0 \\ \text{dilaton :} & \quad \varepsilon_{i\mu\nu} = \frac{1}{\sqrt{8}} (\eta_{\mu\nu} - p_{i\mu} \ell_{i\nu} - \ell_{i\mu} p_{i\nu}) \quad \text{where } p_i \cdot \ell_i = 1 \\ \text{Kalb – Ramond :} & \quad \varepsilon_{i\mu\nu} = -\varepsilon_{i\nu\mu} . \end{aligned} \quad (4)$$

In the amplitude (1), both integrals run over the upper half of the complex plane. As a result, this expression is actually divergent because of the  $SL(2, R)$  invariance of the integrand. We chose to fix this  $SL(2, R)$  invariance by hand (as opposed to introducing diffeomorphism ghosts) since we found it to be a useful intermediate check of our calculations.

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<sup>1</sup>A  $p$ -brane is an object extended in  $p$  spatial directions which then sweeps out a  $p + 1$  dimensional world-volume in ten-dimensional Minkowski-signature spacetime. The special case,  $p = -1$ , refers to a Euclidean instanton.

Separately, the left- and right-moving fields have standard propagators on the upper half plane, *e.g.*,

$$\begin{aligned}\langle X^\mu(z) X^\nu(w) \rangle &= -\eta^{\mu\nu} \log(z-w) \\ \langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\frac{\eta^{\mu\nu}}{z-w} \\ \langle \phi(z) \phi(w) \rangle &= -\log(z-w)\end{aligned}\tag{5}$$

with analogous expressions for the right-movers (see [9, 10]). As a result of the boundary at the real axis, there are also nontrivial correlators between the right- and left-modes as well

$$\langle X^\mu(z) \tilde{X}^\nu(\bar{w}) \rangle = -D^{\mu\nu} \log(z-\bar{w})\tag{6}$$

$$\langle \psi^\mu(z) \tilde{\psi}^\nu(\bar{w}) \rangle = -\frac{D^{\mu\nu}}{z-\bar{w}}\tag{7}$$

$$\langle \phi(z) \tilde{\phi}(\bar{w}) \rangle = -\log(z-\bar{w}) .$$

These propagators have the standard form (5) for the fields satisfying Neumann boundary conditions, while the matrix  $D$  reverses the sign for the fields satisfying Dirichlet conditions, *i.e.*, for  $X^\mu$  and  $\psi^\mu$  for  $\mu = p+1, \dots, 9$  (see eq. (63) in Appendix A). To simplify the calculations, we make the replacements

$$\tilde{X}^\mu(\bar{z}) \rightarrow D^\mu{}_\nu X^\nu(\bar{z}) \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D^\mu{}_\nu \psi^\nu(\bar{z}) \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z})\tag{8}$$

and which allows us to use the standard correlators (5) throughout our calculations, *i.e.*, we extend the fields to the entire complex plane [7]. With these replacements, the vertex operators (2) become

$$\begin{aligned}V_1(z_1, \bar{z}_1) &= \varepsilon_{1\mu\lambda} D^\lambda{}_\nu :V_{-1}^\mu(p_1, z_1): :V_{-1}^\nu(D \cdot p_1, \bar{z}_1): \\ V_2(z_2, \bar{z}_2) &= \varepsilon_{2\mu\lambda} D^\lambda{}_\nu :V_0^\mu(p_2, z_2): :V_0^\nu(D \cdot p_2, \bar{z}_2): \end{aligned}$$

using only the expressions in eq. (3).

It is then straightforward to evaluate the correlation function appearing in the amplitude (1), and to confirm that the result is  $SL(2, R)$  invariant. To fix this invariance, we set  $z_1 = iy$  and  $z_2 = i$ . Introducing the appropriate  $SL(2, R)$  Jacobian,<sup>2</sup>

$$d^2z_1 d^2z_2 \rightarrow 4(1-y^2)dy$$

we are left with a single real integral of the form

$$\begin{aligned}A &= -i\kappa T_p 2^{p_1 \cdot D \cdot p_1 + p_2 \cdot D \cdot p_2 + 1} \int_0^1 dy y^{p_2 \cdot D \cdot p_2} (1-y)^{2p_1 \cdot p_2} (1+y)^{2p_1 \cdot D \cdot p_2} \\ &\quad \times \left[ \frac{1}{1-y^2} a_1 - \frac{(1-y)}{4y(1+y)} a_2 \right]\end{aligned}\tag{9}$$

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<sup>2</sup>Our conventions are such that  $z = x + iy$  and  $d^2z = 2dx dy$ .

where  $a_1$  and  $a_2$  are two kinematic factors depending only on the spacetime momenta and polarization tensors. We have also normalized the amplitude at this point by the introduction of factors of  $\kappa$  and  $T_p$ , the closed string and D-brane coupling constants, respectively.<sup>3</sup> The easiest approach to evaluating these integrals is to transform

$$y = \frac{1 - x^{1/2}}{1 + x^{1/2}}$$

which essentially maps the integral to a radial integral on the unit disk. This transformation has two remarkable effects: First, the momentum-dependent power of two which appears as an overall factor in eq. 9 is cancelled using various momentum identities (see Appendix A). In particular, one has momentum conservation in the directions parallel to the  $p$ -brane,

$$(p_1 + D \cdot p_1 + p_2 + D \cdot p_2)^\mu = 0 \quad . \quad (10)$$

The second effect is that the remaining integrals take the form of Euler beta functions. Hence, the final result may be written as

$$A = -i \frac{\kappa T_p}{2} \frac{\Gamma(-t/2) \Gamma(2q^2)}{\Gamma(1 - t/2 + 2q^2)} \left( 2q^2 a_1 + \frac{t}{2} a_2 \right) \quad (11)$$

where  $t = -(p_1 + p_2)^2 = -2p_1 \cdot p_2$  is the momentum transfer to the  $p$ -brane, and  $q^2 = p_1 \cdot V \cdot p_1 = \frac{1}{2} p_1 \cdot D \cdot p_1$  is the momentum flowing parallel to the world-volume of the brane (see Appendix A). The kinematic factors above are:

$$\begin{aligned} a_1 = & \text{Tr}(\varepsilon_1 \cdot D) p_1 \cdot \varepsilon_2 \cdot p_1 - p_1 \cdot \varepsilon_2 \cdot D \cdot \varepsilon_1 \cdot p_2 - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D \cdot p_1 \\ & - p_1 \cdot \varepsilon_2^T \cdot \varepsilon_1 \cdot D \cdot p_1 - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot p_2 + q^2 \text{Tr}(\varepsilon_1 \cdot \varepsilon_2^T) + \{1 \longleftrightarrow 2\} \end{aligned} \quad (12)$$

$$\begin{aligned} a_2 = & \text{Tr}(\varepsilon_1 \cdot D) (p_1 \cdot \varepsilon_2 \cdot D \cdot p_2 + p_2 \cdot D \cdot \varepsilon_2 \cdot p_1 + p_2 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2) \\ & + p_1 \cdot D \cdot \varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D \cdot p_2 - p_2 \cdot D \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D \cdot p_1 + q^2 \text{Tr}(\varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D) \\ & - q^2 \text{Tr}(\varepsilon_1 \cdot \varepsilon_2^T) - \text{Tr}(\varepsilon_1 \cdot D) \text{Tr}(\varepsilon_2 \cdot D) (q^2 - t/4) + \{1 \longleftrightarrow 2\} \quad . \end{aligned} \quad (13)$$

Our notation is such that *e.g.*,  $p_1 \cdot \varepsilon_2 \cdot \varepsilon_1^T \cdot D \cdot p_1 = p_1^\mu \varepsilon_{2\mu\nu} \varepsilon_1^{\lambda\nu} D_{\lambda\rho} p_1^\rho$ .

From the gamma function factors appearing in eq. (11), we see that the amplitudes contain two infinite series poles<sup>4</sup> corresponding to closed string states in the  $t$ -channel with  $\alpha' m^2 = 4n$ , and to open string states in the  $q^2$ - or  $s$ -channel with  $\alpha' m^2 = n$ , with  $n = 0, 1, 2, \dots$ . As is evident the final amplitude is symmetric under the interchange of the two string states, *i.e.*,  $1 \longleftrightarrow 2$ , despite the asymmetric appearance of the initial integrand

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<sup>3</sup>Here and in the subsequent amplitudes, we omit the Dirac delta-function which imposes momentum conservation in the directions to the  $p$ -brane world-volume, *i.e.*, eq. (10). We have introduced a phase  $-i$  though which corresponds to that of the analogous field theory amplitudes calculated in Minkowski space — see sect. 4.2.

<sup>4</sup>We explicitly restore  $\alpha'$  here. Otherwise our conventions set  $\alpha' = 2$ .

in eq. (1). Another check is that the amplitude satisfies the Ward identities associated with the gauge invariances of these states, *i.e.*, the amplitude vanishes upon substituting  $\varepsilon_{i\mu\nu} \rightarrow p_{i\mu} q_{i\nu}$  or  $q_{i\mu} p_{i\nu}$ , where  $q_i \cdot p_i = 0$ . In the special case that the polarization tensors have non-vanishing components only in directions perpendicular to the world-volume of the  $p$ -brane (*i.e.*, following Appendix A,  $V_\mu^\nu \varepsilon_{i\nu\rho} = 0 = \varepsilon_{i\mu\nu} V^\nu_\rho$ ) the results simplify greatly. In this case, we find for two gravitons

$$A = -i \frac{\kappa T_p}{2} \frac{\Gamma(-t/2)\Gamma(1+2q^2)}{\Gamma(1-t/2+2q^2)} 2q^2 \text{Tr}(\varepsilon_2 \cdot \varepsilon_1)$$

while for two Kalb-Ramond particles,

$$A = -i \frac{\kappa T_p}{2} \frac{\Gamma(-t/2)\Gamma(1+2q^2)}{\Gamma(1-t/2+2q^2)} \left( 4 p_1 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_2 + (t - 2q^2) \text{Tr}(\varepsilon_2 \cdot \varepsilon_1) \right)$$

which agrees with the results of Ref. [6, 7].

The purpose behind our rather lengthy description of the calculations for the NS-NS scattering amplitudes is to compare these calculations to that of an apparently unrelated scattering amplitude, namely, the amplitude for four massless NS vectors in open superstring theory. The latter would be calculated as four vertex operator insertions on the boundary of a disk on which Neumann boundary conditions are imposed. Again the disk can be mapped to the upper half of the complex plane, in which case the amplitude becomes

$$A \simeq \int dx_1 dx_2 dx_3 dx_4 \langle : \zeta_1 \cdot V_{-1}(2k_1, x_1) : : \zeta_2 \cdot V_0(2k_2, x_2) : : \zeta_3 \cdot V_0(2k_3, x_3) : : \zeta_4 \cdot V_{-1}(2k_4, x_4) : \rangle \quad (14)$$

where the vertex operators are written in terms of the same components given in eq. (3) which were used to construct the previous closed string amplitudes.<sup>5</sup> One evaluates the correlation functions using the same standard propagators appearing in eq. (5). In this amplitude (14),  $x_i$  lie on the real axis and the range of integration is  $-\infty \leq x_1 \leq x_2 \leq x_3 \leq x_4 \leq \infty$ . Therefore this expression diverges because of the  $SL(2, R)$  invariance of the integrand. The standard approach to fixing this invariance is to set  $x_1 = 0, x_2 = x, x_3 = 1, x_4 = \infty$ . Here, we make an alternate choice instead setting  $x_1 = -1, x_2 = -x_3, x_3 = x$ , and  $x_4 = 1$ . Introducing the appropriate  $SL(2, R)$  Jacobian,

$$dx_1 dx_2 dx_3 dx_4 \rightarrow (1 - x^2) dx$$

we are left with a single real integral

$$A \simeq \int_0^1 dx x^{4k_2 \cdot k_3} (1-x)^{8k_1 \cdot k_2} (1+x)^{8k_1 \cdot k_3} \left[ \frac{1}{1-x^2} a'_1 - \frac{1-x}{4x(1+x)} a'_2 \right]$$

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<sup>5</sup>Each open string vertex operator is labelled with momentum  $2k_i$  since we maintain our convention that  $\alpha' = 2$  even though this is an open string scattering amplitude.

which has essentially the same form as eq. (9) for the closed string amplitudes. The final result here is well-known [11] and may be written as<sup>6</sup>

$$A(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) = -\frac{1}{2}g^2 \frac{\Gamma(4k_1 \cdot k_2)\Gamma(4k_1 \cdot k_4)}{\Gamma(1 + 4k_1 \cdot k_2 + 4k_1 \cdot k_4)} K(\zeta_1, k_1; \zeta_2, k_2; \zeta_3, k_3; \zeta_4, k_4) \quad . \quad (15)$$

where  $g$  is the open string coupling constant. The (rather lengthy) kinematic factor may be written as

$$\begin{aligned} K = & -16k_2 \cdot k_3 k_2 \cdot k_4 \zeta_1 \cdot \zeta_2 \zeta_3 \cdot \zeta_4 \\ & -4k_1 \cdot k_2 (\zeta_1 \cdot k_4 \zeta_3 \cdot k_2 \zeta_2 \cdot \zeta_4 + \zeta_2 \cdot k_3 \zeta_4 \cdot k_1 \zeta_1 \cdot \zeta_3 + \zeta_1 \cdot k_3 \zeta_4 \cdot k_2 \zeta_2 \cdot \zeta_3 + \zeta_2 \cdot k_4 \zeta_3 \cdot k_1 \zeta_1 \cdot \zeta_4) \\ & + \{1, 2, 3, 4 \rightarrow 1, 3, 2, 4\} + \{1, 2, 3, 4 \rightarrow 1, 4, 3, 2\} \quad . \end{aligned}$$

One can see that this expression displays an infinite set of open string poles in both the  $s$ - and  $t$ -channels with  $\alpha' m^2 = n$  where  $n = 0, 1, 2, \dots$

Now what becomes clear from this discussion is that both calculations involve precisely the same correlation functions and exactly the same integrals after fixing the  $SL(2, R)$  invariance. Thus up to an overall normalization of the coupling constants, both amplitudes are identical. One need only make the following substitution to convert the four vector amplitude in the open superstring theory into a D-brane amplitude for two NS-NS closed superstring states:

$$\begin{aligned} 2k_1^\mu &\rightarrow p_1^\mu & 2k_4^\mu &\rightarrow (D \cdot p_1)^\mu \\ 2k_2^\mu &\rightarrow p_2^\mu & 2k_3^\mu &\rightarrow (D \cdot p_2)^\mu \\ \zeta_{1\mu} \otimes \zeta_{4\nu} &\rightarrow \varepsilon_{1\mu\lambda} D^\lambda{}_\nu \\ \zeta_{2\mu} \otimes \zeta_{3\nu} &\rightarrow \varepsilon_{2\mu\lambda} D^\lambda{}_\nu \quad , \end{aligned} \quad (16)$$

as well as replacing  $g^2 \rightarrow i\kappa T_p$ . We have directly confirmed that these replacements in eq. (15) precisely reproduce the results in eqs. (11-13). It is interesting to note that with these substitutions momentum conservation in the open string amplitude becomes precisely momentum conservation in the directions parallel to the D-brane as in eq. (10). These observations above allow us to easily calculate any Dirichlet two-point amplitudes by simply using the well-known results for four-point open string scattering amplitudes [11], as described in the following section.

### 3 Dirichlet two-point amplitudes

The relation between the four-vector scattering amplitude for the open superstring and the two-point amplitude for scattering of two NS-NS closed superstring states from a Dirichlet

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<sup>6</sup>Here and in the following section, we present the open superstring amplitudes as calculated with the conformal field theory conventions described above — see also Appendix B for spin operators.

brane may seem somewhat surprising. In fact, it might be regarded as an extension of the results in Ref. [12]. There it was shown that closed string amplitudes could be expressed as products of open string amplitudes corresponding to the correlation functions appearing in the independent right- and left-moving sectors along with certain “sewing” factors. The present case is similar except that with the D-brane boundary conditions, the right- and left-movers are naturally “sewn” together in a single open string amplitude. Given this idea, it is straightforward to write down all of the remaining Dirichlet brane two-point amplitudes using the well-known four-particle open superstring amplitudes involving massless spinors, as well as vectors. These amplitudes may all be expressed in the form [11]

$$A(1, 2, 3, 4) = -\frac{1}{2}g^2 \frac{\Gamma(4k_1 \cdot k_2)\Gamma(4k_1 \cdot k_4)}{\Gamma(1 + 4k_1 \cdot k_2 + 4k_1 \cdot k_4)} K(1, 2, 3, 4) \quad . \quad (17)$$

The various kinematic factors are then given by

$$K(u_1, u_2, u_3, u_4) = -2k_1 \cdot k_2 \bar{u}_2 \gamma^\mu u_3 \bar{u}_1 \gamma_\mu u_4 + 2k_1 \cdot k_4 \bar{u}_1 \gamma^\mu u_2 \bar{u}_4 \gamma_\mu u_3 \quad (18)$$

$$K(u_1, \zeta_2, \zeta_3, u_4) = 2i\sqrt{2}k_1 \cdot k_4 \bar{u}_1 \gamma \cdot \zeta_2 \gamma \cdot (k_3 + k_4) \gamma \cdot \zeta_3 u_4 \quad (19)$$

$$-4i\sqrt{2}k_1 \cdot k_2 (\bar{u}_1 \gamma \cdot \zeta_3 u_4 k_3 \cdot \zeta_2 - \bar{u}_1 \gamma \cdot \zeta_2 u_4 k_2 \cdot \zeta_3 - \bar{u}_1 \gamma \cdot k_3 u_4 \zeta_2 \cdot \zeta_3)$$

$$K(u_1, \zeta_2, u_3, \zeta_4) = -2i\sqrt{2}k_1 \cdot k_4 \bar{u}_1 \gamma \cdot \zeta_2 \gamma \cdot (k_3 + k_4) \gamma \cdot \zeta_4 u_3 \quad (20)$$

$$-2i\sqrt{2}k_1 \cdot k_2 \bar{u}_1 \gamma \cdot \zeta_4 \gamma \cdot (k_2 + k_3) \gamma \cdot \zeta_2 u_3 \quad .$$

In translating these results to Dirichlet two-point amplitudes, schematically one associates (some cyclic permutation of)  $(1, 2, 3, 4)$  in the open string amplitude with  $(1_L, 2_L, 2_R, 1_R)$  in the closed string amplitude, where here the subscripts  $L$  and  $R$  denote the left- and right-moving components of the closed string states. Having chosen a particular ordering, the translation of the NS sector contributions between the open and closed string amplitudes remains the same as in eq. (16). The interesting feature is the appearance of a factor of  $D_{\mu\nu}$  in the momenta and polarization tensors of the right-moving contributions. An analogous spinor matrix  $M_{AB}$  also appears in the right-moving Ramond sector contributions — see Appendix B. The D-brane scattering amplitudes then take a universal form

$$A(1, 2) = -i \frac{\kappa T_p}{2} \frac{\Gamma(-t/2)\Gamma(2q^2)}{\Gamma(1 - t/2 + 2q^2)} K(1, 2) \quad (21)$$

where as before  $t$  is the momentum transfer to the  $p$ -brane, and  $q^2$  is the momentum flowing parallel to the world-volume of the brane — see Appendix A. For later discussions, it is useful to divide the kinematic factor as

$$K(1, 2) = 2q^2 a_1(1, 2) + \frac{t}{2} a_2(1, 2) \quad (22)$$

as was done in eq. (11). Then  $a_1(1, 2)$  will be essentially the residue of the massless  $t$ -channel pole, which will become important for the analysis in sect. 4.2. Now, it simply remains to translate the kinematic factors (18)-(20) in the appropriate way.



### 3.1 R-R boson amplitude

The simplest case is using eqs. (17) and (18) to calculate the amplitude describing two R-R states scattering from a Dirichlet brane. The latter amplitude would be written as

$$A \simeq \int d^2 z_1 d^2 z_2 \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2) \rangle$$

where the vertex operators are

$$V_i(z_i, \bar{z}_i) = (P_- \Gamma_{i(n)})^{AB} :V_{-1/2 A}(p_i, z_i) : \tilde{V}_{-1/2 B}(p_i, \bar{z}_i) : \quad . \quad (23)$$

The holomorphic components above are given by

$$V_{-1/2 A}(p_i, z_i) = e^{-\phi(z_i)/2} S_A(z_i) e^{ip_i \cdot X(z_i)} \quad (24)$$

and the antiholomorphic components have the same form, but with the left-moving fields replaced by their right-moving counterparts. As before we use eq. (8) to replace  $\tilde{X}^\mu$  and  $\tilde{\phi}$  in  $\tilde{V}_{-1/2 B}$ . Similarly, the right-moving spin field is replaced using [7] (see also [13])

$$\tilde{S}_A(\bar{z}) \rightarrow M_A^B S_B(\bar{z}) \quad (25)$$

where  $M_{AB}$  is defined in Appendix B. With this replacement, only standard correlators of the spin fields [14, 9, 10] appear in the subsequent calculations. We have explicitly included the chiral projection operator  $P_- = (1 - \gamma_{11})/2$  in vertex operator (23), so that our calculations are always made with the full  $32 \times 32$  Dirac matrices of ten dimensions. We have also defined

$$\Gamma_{i(n)} = \frac{a_n}{n!} F_{\mu_1 \dots \mu_n}^i \gamma^{\mu_1} \dots \gamma^{\mu_n} \quad . \quad (26)$$

With our choice of conventions (see Appendix B), we must introduce the factor  $a_n = i$  for the  $n = 2$  and 4 fields in the type IIA theory, while  $a_n = 1$  for  $n = 1, 3$  and 5 in the type IIB theory. In eq. (26),  $F_{\mu_1 \dots \mu_n}^i$  is the linearized  $n$ -form field strength with

$$\begin{aligned} F_{\mu_1 \dots \mu_n}^i &= i n p_{i[\mu_1} \varepsilon_{i\mu_2 \dots \mu_n]} \\ &= i p_{i\mu_1} \varepsilon_{i\mu_2 \dots \mu_n} \pm \text{cyclic permutations} \end{aligned} \quad (27)$$

where  $p_i^2 = 0$  and  $p_i^\mu \varepsilon_{i\mu\mu_3 \dots \mu_n} = 0$ . Hence the appropriate substitutions for the open string amplitude (18) to derive the Dirichlet amplitude are

$$\begin{aligned} 2k_1^\mu &\rightarrow p_1^\mu & 2k_4^\mu &\rightarrow (D \cdot p_1)^\mu \\ 2k_2^\mu &\rightarrow p_2^\mu & 2k_3^\mu &\rightarrow (D \cdot p_2)^\mu \\ u_{1A} \otimes u_{4B} &\rightarrow (P_- \Gamma_{1(n)} M)_{AB} \\ u_{2A} \otimes u_{3B} &\rightarrow (P_- \Gamma_{2(m)} M)_{AB} \quad . \end{aligned} \quad (28)$$

The resulting kinematic factor is:

$$\begin{aligned}
K_{R-R,R-R} &= -q^2 \text{Tr}(P_- \Gamma_{1(n)} M \gamma_\mu C^{-1} M^T \Gamma_{2(m)}^T C \gamma^\mu) \\
&\quad + \frac{t}{4} \text{Tr}(P_- \Gamma_{1(n)} M \gamma_\mu) \text{Tr}(P_- \Gamma_{2(m)} M \gamma^\mu)
\end{aligned} \tag{29}$$

This scattering amplitude was previously calculated in Ref. [7]. In their results,  $a_1$  (*i.e.*, the first term above) does not quite appear with the same form as here. However, in sect. 4.2 with an explicit evaluation of the trace in  $a_1$ , we will show that our results are identical to those of [7].

### 3.2 NS-NS and R-R amplitude

The next case is calculating the amplitude describing one R-R and one NS-NS state scattering from a Dirichlet brane using eqs. (17) and (19). The appropriate substitutions to derive the Dirichlet amplitude are already derived for the previous amplitudes in eqs. (16) and (28)

$$\begin{aligned}
2k_1^\mu &\rightarrow p_1^\mu & 2k_4^\mu &\rightarrow (D \cdot p_1)^\mu \\
2k_2^\mu &\rightarrow p_2^\mu & 2k_3^\mu &\rightarrow (D \cdot p_2)^\mu \\
u_{1A} \otimes u_{4B} &\rightarrow (P_- \Gamma_{1(n)} M)_{AB} \\
\zeta_{2\mu} \otimes \zeta_{3\nu} &\rightarrow \varepsilon_{2\mu\lambda} D^\lambda{}_\nu
\end{aligned} .$$

The resulting kinematic factor is then:

$$\begin{aligned}
K_{R-R,NS-NS} &= i \frac{q^2}{\sqrt{2}} \text{Tr}[P_- \Gamma_{1(n)} M \gamma^\nu \gamma \cdot (p_1 + p_2) \gamma^\mu] (\varepsilon_2 \cdot D)_{\mu\nu} \\
&\quad - i \frac{t}{2\sqrt{2}} \left[ \text{Tr}(P_- \Gamma_{1(n)} M \gamma \cdot D \cdot \varepsilon_2^T \cdot D \cdot p_2) - \text{Tr}(P_- \Gamma_{1(n)} M \gamma \cdot \varepsilon_2 \cdot D \cdot p_2) \right. \\
&\quad \left. - \text{Tr}(P_- \Gamma_{1(n)} M \gamma \cdot D \cdot p_2) \text{Tr}(\varepsilon_2 \cdot D) \right]
\end{aligned} \tag{30}$$

Just as for eq. (11), this result inherits the gauge invariance of the NS states in the open string amplitude as the closed string gauge invariance for the NS-NS state. This amplitude will be of particular interest in the following section for determining the background R-R fields in sect. 4.

### 3.3 R-NS fermion amplitude

We can also calculate the fermionic scattering amplitudes as well. Making the alternative match in eq. (19) which identifies  $(4, 1, 2, 3)$  with  $(1_L, 2_L, 2_R, 1_R)$  results in a scattering amplitude describing two states in the R-NS sector of the closed string. The latter amplitude would be written as

$$A \simeq \int d^2 z_1 d^2 z_2 \langle V_1(z_1, \bar{z}_1) V_2(z_2, \bar{z}_2) \rangle$$

where the vertex operators are

$$\begin{aligned} V_1(z_1, \bar{z}_1) &= P_-^{AB} \psi_{1\mu B} D^\mu{}_\nu :V_{-1/2 A}(p_1, z_1): :V_{-1}^\nu(D \cdot p_1, \bar{z}_1): \\ V_2(z_2, \bar{z}_2) &= P_-^{AB} \psi_{2\mu B} D^\mu{}_\nu :V_{-1/2 A}(p_2, z_2): :V_0^\nu(D \cdot p_1, \bar{z}_1): \end{aligned}$$

where  $V_0$  and  $V_{-1}$  are given eq. (3), while  $V_{-1/2}$  appears in eq. (24). Here we have already replaced the right-moving modes by the appropriate expressions as in eqs. (8) and (25). Again we have explicitly included the chiral projection operator  $P_- = (1 - \gamma_{11})/2$ , even though the polarization tensors implicitly satisfy  $P_- \psi_{i\mu} = \psi_{i\mu}$ . Further the momenta and polarizations satisfy

$$p_i^2 = 0 \quad (\gamma \cdot p_i)_A{}^B \psi_{i\mu B} = 0 \quad p_i^\mu \psi_{i\mu B} = 0 \quad .$$

The physical spinor states are represented with

$$\begin{aligned} \text{gravitino :} & \quad \psi_{i\mu A} & \text{where } (\gamma^\mu)_A{}^B \psi_{i\mu B} = 0 \\ \text{dilatinio :} & \quad \psi_{i\mu A} = (\gamma_\mu)_A{}^B \chi_{iB} & \text{where } (\gamma \cdot p_i)_A{}^B \chi_{iB} = 0 \quad . \end{aligned}$$

The appropriate substitutions to derive the Dirichlet amplitude are then

$$\begin{aligned} 2k_4^\mu &\rightarrow p_1^\mu & 2k_3^\mu &\rightarrow (D \cdot p_1)^\mu \\ 2k_1^\mu &\rightarrow p_2^\mu & 2k_2^\mu &\rightarrow (D \cdot p_2)^\mu \\ u_{4A} \otimes \zeta_{3\mu} &\rightarrow (P_- \psi_1 \cdot D)_{\mu A} \\ u_{1A} \otimes \zeta_{2\mu} &\rightarrow (P_- \psi_2 \cdot D)_{\mu A} \quad . \end{aligned}$$

The resulting kinematic factor is then:

$$\begin{aligned} K_{R-NS, R-NS} &= i\sqrt{2} q^2 (\psi_2 \cdot p_1 \gamma \cdot D \cdot P_- \psi_1 - \psi_2 \cdot D \cdot \gamma p_2 \cdot P_- \psi_1 \\ &\quad - \psi_2^\mu \gamma \cdot D \cdot p_1 P_- \psi_{1\mu}) \\ &\quad + i \frac{t}{4\sqrt{2}} (\psi_2 \cdot D \cdot \gamma \gamma \cdot (p_1 + D \cdot p_1) \gamma \cdot D \cdot P_- \psi_1) \end{aligned} \quad (31)$$

These amplitudes vanish with the substitution  $\psi_{i\mu A} \rightarrow p_{i\mu} \chi_A$  where  $(\gamma \cdot p_i)_A{}^B \chi_B = 0$ . This Ward identity for the gauged supersymmetry in the closed superstring theory is again naturally inherited from the vector gauge invariance of the corresponding open superstring amplitude.

### 3.4 NS-R fermion amplitude

Identifying the cyclic permutation (2, 3, 4, 1) in eq. (19) with  $(1_L, 2_L, 2_R, 1_R)$  in the Dirichlet amplitude results in a scattering amplitude for two spinors in the NS-R sector. In this case, the previous discussion motivates the substitutions

$$\begin{aligned} 2k_2^\mu &\rightarrow p_1^\mu & 2k_1^\mu &\rightarrow (D \cdot p_1)^\mu \\ 2k_3^\mu &\rightarrow p_2^\mu & 2k_4^\mu &\rightarrow (D \cdot p_2)^\mu \\ \zeta_{3\mu} \otimes u_{4A} &\rightarrow (MP_\pm \psi_2)_{\mu A} \\ \zeta_{2\mu} \otimes u_{1A} &\rightarrow (MP_\pm \psi_1)_{\mu A} \end{aligned}$$

where  $P_{+(-)}$  is chosen in the type IIA(b) superstring theory.<sup>7</sup> The resulting kinematic factor is then:

$$\begin{aligned}
K_{NS-R, NS-R} = & i\sqrt{2}q^2(\psi_1 \cdot p_2 M^{-1} \gamma \cdot M P_{\pm} \psi_1 - \psi_2 M^{-1} \cdot \gamma M P_{\pm} p_1 \cdot \psi_2 \\
& - \psi_1^{\mu} M^{-1} \gamma \cdot p_2 M P_{\pm} \psi_{2\mu}) \\
& + i\frac{t}{4\sqrt{2}}(\psi_1 M^{-1} \cdot \gamma \gamma \cdot (p_2 + D \cdot p_2) \gamma \cdot M P_{\pm} \psi_2)
\end{aligned} \tag{32}$$

where  $M^{-1} = C^{-1} M^T C$ . This kinematic factor again satisfies the appropriate Ward identities as in eq. (31). Using identities such as  $(M \gamma^{\mu}) = D_{\mu\nu}(\gamma_{\nu} M)$  (see Appendix B), one can show the two kinematic factors in eqs. (31) and (32) are in fact identical (up to a sign and the chiral projection in the type IIA theory).

### 3.5 R-NS and NS-R amplitude

Finally eq. (20) yields the kinematic factor for the scattering two fermions with one each in the NS-R and R-NS sectors. With

$$\begin{aligned}
2k_1^{\mu} &\rightarrow p_1^{\mu} & 2k_4^{\mu} &\rightarrow (D \cdot p_1)^{\mu} \\
2k_2^{\mu} &\rightarrow p_2^{\mu} & 2k_3^{\mu} &\rightarrow (D \cdot p_2)^{\mu} \\
u_{1A} \otimes \zeta_{4\mu} &\rightarrow (P_- \psi_1 \cdot D)_{\mu A} \\
\zeta_{2\mu} \otimes u_{3A} &\rightarrow (M P_{\pm} \psi_2)_{\mu A} \quad .
\end{aligned}$$

one finds that

$$\begin{aligned}
K_{R-NS, NS-R} = & -i\frac{q^2}{\sqrt{2}}(\psi_1 P_+ \cdot D_{\nu} \gamma^{\mu} \gamma \cdot (p_1 + p_2) \gamma^{\nu} M \psi_{2\mu}) \\
& -i\frac{t}{4\sqrt{2}}(\psi_1 P_+ \cdot D \cdot \gamma \gamma \cdot (p_2 + D \cdot p_2) \gamma \cdot M \psi_2) \quad .
\end{aligned}$$

Note that the chiral projections of the spinors are consistent *i.e.*, allow for a nonvanishing amplitude, because of the form of  $M$  given in eq. (68) and the fact that  $p$  is even and odd in the type IIA and IIB theories, respectively.

## 4 Background fields

In the context of various field theories, many different  $p$ -brane solutions have been constructed describing extended objects of different dimensions. (see [15] as well as [16] and references therein). Typically these solutions involve a  $(n-1)$ -form potential coupled to gravity and a scalar field, *i.e.*, the dilaton. For a potential of form degree  $(n-1)$  in  $d$

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<sup>7</sup>Our convention is that the Ramond ground-state in the left-moving sector has negative chirality in both theories. For the right-movers, the opposite (same) chirality is chosen in the type IIA(b) theory.

spacetime dimensions, one naturally finds two dual classes of  $p$ -branes. The first with  $p = n - 2$  carries an “electric” charge of the  $n$ -form field strength. The dual object with  $p = d - n - 2$  carries an analogous “magnetic” charge.

In the following section, we review some of these  $p$ -brane solutions. Then in sect. 4.2, we examine our D-brane scattering amplitudes for massless  $t$ -channel poles. These poles correspond to the interactions produced by the long range background fields around the Dirichlet  $p$ -branes. Finally in sect. 4.3, we make a detailed comparison of the  $Dp$ -brane fields with those in the field theory solutions.

## 4.1 Extremal $p$ -branes

We begin with an action in  $d$  dimensions

$$\hat{I} = \frac{1}{16\pi\hat{G}_N} \int d^d x \sqrt{-\hat{g}} \left[ \hat{R}(\hat{g}) - \frac{\hat{\gamma}}{2} (\nabla\hat{\phi})^2 - \frac{1}{2n!} e^{-\hat{a}\hat{\gamma}\hat{\phi}} \hat{F}_{(n)}^2 \right] \quad (33)$$

describing a potential  $\hat{A}$  coupled to gravity and a dilaton. The field strength  $\hat{F}_{(n)}$  is an  $n$ -form given by  $\hat{F}_{(n)} = d\hat{A}$ , and  $\hat{\gamma} = 2/(d-2)$  is a convenient normalization factor. The focus of our discussion will be solutions describing static, isotropic and extremal  $p$ -branes. Here, “isotropic” means that the solutions are Lorentz invariant in the directions parallel to the world-volume of the  $p$ -brane, while “extremal” requires a zero-force condition is satisfied, *i.e.*, there is a precise cancellation of the static forces between the  $p$ -branes generated by the dilaton, form-field and graviton. In the case that the action (33) can be embedded as part of a supersymmetric theory, extremality becomes the condition that the  $p$ -brane solutions themselves are supersymmetric [16]. It is because one expects both properties to hold for the configurations corresponding to D-branes that we restrict the following discussion to this class of solutions.

We begin with an electrically charged  $p$ -brane solution with  $p = n - 2$ . The metric takes the following form:

$$d\hat{s}^2 = H^{2\alpha}(\vec{x}) (-dt^2 + d\vec{y}^2) + H^{2\beta}(\vec{x}) d\vec{x}^2 \quad . \quad (34)$$

Here the time,  $t$ , and the  $p = n - 2$  spatial coordinates,  $y^a$ , run parallel to the surface of the brane, while the orthogonal subspace is covered by the  $\bar{p} = d - n + 1$  coordinates,  $x^i$ . The  $(n - 1)$ -form potential and the dilaton may be written as

$$\hat{A} = \pm\sqrt{2\sigma} H(\vec{x})^{-1} \epsilon^v \quad \text{and} \quad e^{-\hat{\phi}} = H(\vec{x})^\tau \quad (35)$$

where  $\epsilon^v = dt dy^1 \cdots dy^p$  is the volume form in the subspace parallel to the  $p$ -brane world-volume – see Appendix A. For later discussion, it is also convenient to present the field strength  $F_{(n)}$ . Given the potential in eq. (35), the latter is given by

$$\hat{F}_{(n)} = \mp\sqrt{2\sigma} H^{-2} \partial_j H dx^j \wedge \epsilon^v \quad .$$

With an appropriate choice of exponents, namely,

$$\begin{aligned}\alpha &= -\frac{\bar{p}-2}{(p+1)(\bar{p}-2)+\hat{a}^2} & \beta &= \frac{p+1}{(p+1)(\bar{p}-2)+\hat{a}^2} \\ \sigma &= \frac{d-2}{(p+1)(\bar{p}-2)+\hat{a}^2} & \tau &= \hat{a}\sigma\end{aligned}\quad (36)$$

one finds that  $H$  should satisfy the flat-space Laplace's equation in the transverse space, *i.e.*,  $\delta^{ij}\partial_i\partial_j H = 0$ . Extremal  $p$ -branes are constructed by introducing  $\delta$ -function sources in the latter equation. For a single  $p$ -brane solution as will be of interest in the following, we choose  $H = 1 + \mu G(|\vec{x}|/\ell)$  with

$$G = \begin{cases} \frac{1}{\bar{p}-2} (\ell/|\vec{x}|)^{\bar{p}-2} & \text{for } \bar{p} \geq 3 \\ -\log(|\vec{x}|/\ell) & \text{for } \bar{p} = 2 \\ -|\vec{x}|/\ell & \text{for } \bar{p} = 1 \end{cases} \quad (37)$$

Here  $\ell$  is an arbitrary length and  $\mu$  is some dimensionless constant. We will consider the latter constant to be small, *i.e.*,  $\mu \ll 1$ , so that we may treat the nontrivial part of the solutions as a perturbation of flat empty space. Certainly this is valid in the region  $|\vec{x}| \gg \ell$  for  $\bar{p} \geq 3$ , but only a formal expansion for  $\bar{p} = 2$  and 1. Note that these solutions may be extended to an instanton with  $p = -1$  by using a euclidean metric without  $t$  or  $y^a$ , and having  $\hat{A}$  be a 0-form or scalar [17, 7].

One can also construct a magnetically charged  $p$ -brane solution with  $p = d - n - 2$ . With this choice of  $p$  and hence  $\bar{p} = n + 1$ , the metric remains as given in eq. (34). The dilaton takes the same form as in eq. (35). but with the opposite sign, *i.e.*,  $e^{+\hat{\phi}} = H(\vec{x})^\tau$ . Finally the form field is magnetic with nonvanishing components in the transverse subspace. The latter is conveniently expressed in terms of the field strength as

$$\hat{F}_{(n)} = \mp \sqrt{2\sigma} \partial_j H i_{\hat{x}^j} \epsilon^n$$

where  $i_{\hat{x}^j}$  denotes the interior product with a unit vector pointing in the  $x^j$  direction. Also  $\epsilon^n = dx^1 \cdots dx^{\bar{p}}$  is the volume form in the subspace orthogonal to the  $p$ -brane. With these choices, the numerical factors (36) are left unchanged when written in terms of  $p$ ,  $\bar{p}$  and  $d$ .

The important feature of these solutions for the purposes of comparison with the string scattering amplitudes will be the behavior of fields in the asymptotic region, *i.e.*,  $|\vec{x}| \rightarrow \infty$ . Expanding with  $\mu \ll 1$ , the asymptotic metric is essentially flat with

$$\begin{aligned}\hat{h}_{\mu\nu} &\equiv \hat{g}_{\mu\nu} - \eta_{\mu\nu} \\ &\simeq 2\mu G(|\vec{x}|/\ell) \text{diag}(-\alpha, \alpha, \dots, \alpha, \beta, \dots, \beta) \quad .\end{aligned}\quad (38)$$

For the “electric”  $p$ -brane with  $p = n - 2$ , one has

$$\begin{aligned}\hat{\phi} &\simeq -\hat{a}\sigma\mu G(|\vec{x}|/\ell) \\ \hat{F}_{(n)} &\simeq \mp \sqrt{2\sigma}\mu \partial_j G(|\vec{x}|/\ell) dx^j \wedge \epsilon^v \quad .\end{aligned}\quad (39)$$

Similarly for the “magnetic”  $p$ -brane with  $p = d - n - 2$ , one finds

$$\begin{aligned}\hat{\phi} &\simeq +\hat{a}\sigma\mu G(|\vec{x}|/\ell) \\ \hat{F}_{(n)} &\simeq \mp\sqrt{2\sigma}\mu\partial_j G(|\vec{x}|/\ell) i_{\hat{x}^j}\epsilon^n.\end{aligned}\quad (40)$$

Given these results, it is straightforward to derive some of the physical quantities which characterize these solutions. The ADM mass per unit  $p$ -volume is defined as [18]:

$$M_p = \frac{1}{16\pi\hat{G}_N} \oint \sum_{i=1}^{\bar{p}} n^i \left[ \sum_{j=1}^{\bar{p}} (\partial_j \hat{h}_{ij} - \partial_i \hat{h}_{jj}) - \sum_{a=1}^p \partial_i \hat{h}_{aa} \right] r^{\bar{p}-1} d\Omega$$

where  $n^i$  is a radial unit vector in the transverse subspace, and  $\hat{G}_N$  is the Newton’s constant appearing in the action (33). Given eqs. (36), (37) and (38), one finds

$$M_p = \frac{\sigma}{8\pi\hat{G}_N} \mathcal{A}_{\bar{p}-1} \mu \ell^{\bar{p}-2} \quad (41)$$

where  $\mathcal{A}_{\bar{p}-1} = 2\pi^{\bar{p}}/\Gamma(\bar{p}/2)$  is the area of a unit  $(\bar{p}-1)$ -sphere. The electric form charge is given by [16]

$$\begin{aligned}Q_E &= \frac{1}{\sqrt{16\pi\hat{G}_N}} \oint * \hat{F}_{(n)} \\ &= \pm(-)^{p\bar{p}} \sqrt{\frac{\sigma}{8\pi\hat{G}_N}} \mathcal{A}_{\bar{p}-1} \mu \ell^{\bar{p}-2}.\end{aligned}\quad (42)$$

Similarly the magnetic form charge is give by

$$\begin{aligned}Q_M &= \frac{1}{\sqrt{16\pi\hat{G}_N}} \oint \hat{F}_{(n)} \\ &= \pm \sqrt{\frac{\sigma}{8\pi\hat{G}_N}} \mathcal{A}_{\bar{p}-1} \mu \ell^{\bar{p}-2}.\end{aligned}\quad (43)$$

## 4.2 Massless $t$ -channel poles

Recall that in the scattering amplitudes, the momentum transfer to the D-brane is  $t = -(p_1 + p_2)^2$ . Given the general form of the string amplitudes in eqs. (21-22), one can expand these amplitudes as an infinite sum of terms reflecting the infinite tower of closed string states that couple to the D-brane in the  $t$ -channel (*i.e.*, terms with poles at  $\alpha' t = \alpha' m^2 = 4n$  with  $n = 0, 1, 2, \dots$ ). For low momentum transfer, *i.e.*,  $\alpha' t \ll 1$ , the first term representing the exchange of massless string states dominates. In this case, eqs. (21 – 22) reduce to

$$A \simeq i\kappa T_p \frac{a_1}{t}.\quad (44)$$

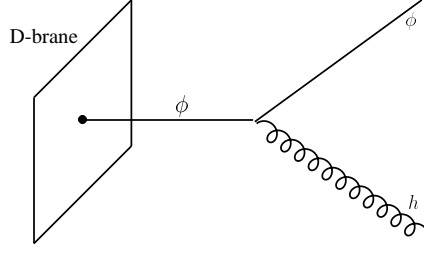


Figure 1: Feynman diagram for graviton-dilaton scattering from a D-brane

One can reproduce these long-range interactions with a calculation in the low energy effective field theory in which  $Dp$ -brane source terms are added to the field theory action. Alternatively, one can think of these amplitudes as representing the interaction of the external string states with the long range background fields generated by the  $Dp$ -brane (see *e.g.*, [19]). Since ultimately we are interested in determining these background fields, we will make use of both interpretations of these amplitudes (44).

The NS-NS sector is common to both type II superstring theories, and so the same low energy effective action describes the graviton, dilaton and Kalb-Ramond fields in both theories. The latter may be written as

$$I_{NS-NS} = \int d^{10}x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - \frac{1}{2} (\nabla\phi)^2 - \frac{3}{2} H^2 e^{-\sqrt{2}\kappa\phi} \right] \quad (45)$$

where  $H = \frac{1}{3}(\partial_\alpha B_{\mu\nu} + \partial_\nu B_{\alpha\mu} + \partial_\mu B_{\nu\alpha})$ . Given this low energy effective action, one can calculate the different propagators, interactions, and subsequently scattering amplitudes for these three massless NS-NS particles. In doing so, one defines the graviton field by  $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$ . One can verify that this action (45) correctly describes the three-point scattering of NS-NS states on the sphere.

To consider the scattering of these particles from a  $Dp$ -brane, we would supplement the low energy action with source terms for the brane as follows:

$$I_{source} = \int d^{10}x [S_B^{\mu\nu} B_{\mu\nu} + S_\phi \phi + S_h^{\mu\nu} h_{\mu\nu}] \quad . \quad (46)$$

We did not make the effort here to extend these source terms in a covariant manner [20, 21], since this will be irrelevant for our leading order calculations. Also note that at least to leading order,  $S_B$ ,  $S_\phi$  and  $S_h$  above will be  $\delta$ -function sources which are only nonvanishing at  $x^i = 0$  using the coordinates of the previous section. We begin by determining the dilaton source  $S_\phi$ . To this end, we consider a scattering process in which an external dilaton is converted to a graviton. The Feynman diagram corresponding to this process appears in figure (1). Examining the low energy action (45), one finds that the only relevant three-point interaction is one graviton coupling to two dilatons through the dilaton kinetic term. Thus in figure (1), the only particle appearing in the  $t$ -channel is the dilaton, and hence



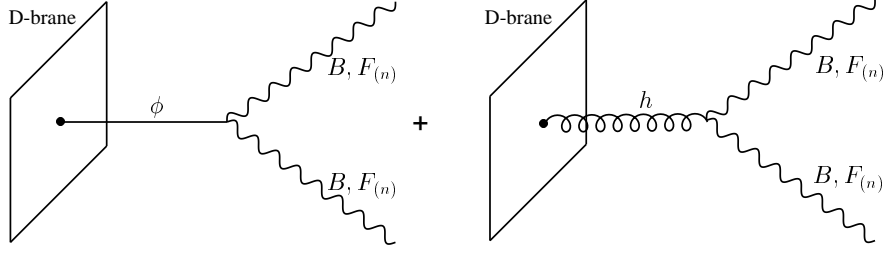


Figure 2: Feynman diagrams for scattering of two (NS-NS or R-R) antisymmetric tensor states from a D-brane

this amplitude will uniquely determine  $S_\phi$ . The field theory amplitude may be written

$$A'_{h\phi} = i\tilde{S}_\phi(k) \tilde{G}_\phi(k^2) \tilde{V}_{h\phi\phi}(\varepsilon_1, p_1, p_2)$$

where  $\tilde{S}_\phi(k)$  is the Fourier transform of the dilaton source,  $\tilde{G}_\phi(k^2) = -i/k^2$  is the dilaton's Feynman propagator, and

$$\tilde{V}_{h\phi\phi} = -i 2\kappa p_2 \cdot \varepsilon_1 \cdot k$$

is the vertex factor for the graviton-dilaton-dilaton interaction. Here,  $\varepsilon_1$  is the graviton polarization tensor, and  $k^\mu = -(p_1 + p_2)^\mu$  is the  $t$ -channel momentum. (We have not included in  $A'_{h\phi}$  a  $\delta$ -function which imposes momentum conservation in the directions parallel to the  $Dp$ -brane.) The analogous string amplitude  $A_{h\phi}$  is constructed from eq. (11) by inserting the appropriate external polarization tensors from eq. (4). Comparing  $A'_{h\phi}$  with the massless  $t$ -channel pole in  $A_{h\phi}$ , one finds agreement by setting

$$\tilde{S}_\phi(k) = -\frac{T_p}{4\sqrt{2}} (2 + \text{Tr}(D)) = -\frac{T_p}{2\sqrt{2}} (p - 3) \quad (47)$$

where we used that  $\text{Tr}(D) = 2p - 8$ . Note that the source is a constant independent of  $k$  in agreement with the expectation that the position space source in eq. (46) is a  $\delta$ -function in the transverse directions.

The graviton source  $S_h$  can be determined from either  $h$ - $h$  or  $B$ - $B$  scattering from the Dirichlet brane. In the first, massless  $t$ -channel exchange is mediated by only a graviton, while in the second, both a graviton and dilaton propagate in the  $t$ -channel. We will consider the scattering of antisymmetric tensors here because the relevant three-point interactions are much simpler. The Feynman diagrams for  $B$ - $B$  scattering are shown in figure (2). The corresponding amplitude is

$$A'_{BB} = i\tilde{S}_h^{\mu\nu}(k) (\tilde{G}_h)_{\mu\nu,\lambda\rho}(k^2) (\tilde{V}_{hBB})^{\lambda\rho} + i\tilde{S}_\phi(k) \tilde{G}_\phi(k^2) \tilde{V}_{\phi BB} \quad (48)$$

where the graviton propagator (in Feynman-like gauge — see *e.g.*, [22]) and the three-point interactions are given by

$$(\tilde{G}_h)_{\mu\nu,\lambda\rho} = -\frac{i}{2} \left( \eta_{\mu\lambda}\eta_{\nu\rho} + \eta_{\mu\rho}\eta_{\nu\lambda} - \frac{1}{4}\eta_{\mu\nu}\eta_{\lambda\rho} \right) \frac{1}{k^2}$$

$$\begin{aligned}
(\tilde{V}_{hBB})^{\lambda\rho} &= -i 2\kappa \left( \frac{1}{2} \left( p_1 \cdot p_2 \eta^{\lambda\rho} - p_1^\lambda p_2^\rho - p_1^\rho p_2^\lambda \right) \text{Tr}(\varepsilon_1 \cdot \varepsilon_2) \right. \\
&\quad \left. - p_1 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_2 \eta^{\lambda\rho} + 2 p_1^{(\lambda} \varepsilon_2^{\rho)} \cdot \varepsilon_1 \cdot p_2 + 2 p_2^{(\lambda} \varepsilon_1^{\rho)} \cdot \varepsilon_2 \cdot p_1 \right. \\
&\quad \left. + 2 p_1 \cdot \varepsilon_2^{(\lambda} \varepsilon_1^{\rho)} \cdot p_2 - p_1 \cdot p_2 (\varepsilon_1^\lambda \cdot \varepsilon_2^\rho + \varepsilon_2^\lambda \cdot \varepsilon_1^\rho) \right) \\
\tilde{V}_{\phi BB} &= -i\sqrt{2}\kappa (2p_1 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_2 - p_1 \cdot p_2 \text{Tr}(\varepsilon_1 \cdot \varepsilon_2))
\end{aligned}$$

where our notation is such that  $\text{Tr}(\varepsilon_1 \cdot \varepsilon_2) = \varepsilon_1^{\mu\nu} \varepsilon_{2\nu\mu}$ ,  $p_1 \cdot \varepsilon_2^\lambda = p_{1\delta} \varepsilon_2^{\delta\lambda}$  and  ${}^\rho \varepsilon_1 \cdot p_2 = \varepsilon_1^{\rho\delta} p_{2\delta}$ . Now again we must compare this result with the massless  $t$ -channel pole in the string amplitude (11) with an appropriate choice of polarization tensors. Unravelling  $\tilde{S}^{\mu\nu}$  from eq. (48) is simplified by noting that the only symmetric two-tensor available is

$$\tilde{S}_h^{\mu\nu}(k^2) = a(k^2) V^{\mu\nu} + b(k^2) N^{\mu\nu} + c(k^2) k^\mu k^\nu \quad (49)$$

given the symmetries of the scattering process. Here,  $V$  ( $N$ ) is the metric in the subspace parallel (orthogonal) to the world-volume of the  $Dp$ -brane — see Appendix A. Now comparing  $A'_{BB}$  with the massless  $t$ -channel pole in the string amplitude  $A_{BB}$  fixes  $a$ ,  $b$  and  $c$  to be constants with  $b = c = 0$  leaving

$$\tilde{S}_h^{\mu\nu} = -T_p V^{\mu\nu} \quad (50)$$

This source is essentially the (Fourier transform of) the  $Dp$ -brane's stress energy tensor, (*i.e.*,  $\tilde{S}_h^{\mu\nu} = \kappa \tilde{T}^{\mu\nu}$ ). Hence we see that  $T_p$  is essentially the D-brane tension, and that as expected there is only stress energy in the world-volume directions. As a cross check, we have calculated  $\tilde{S}_h^{\mu\nu}$  from graviton-graviton scattering, and found the same result. One can also calculate dilaton-dilaton scattering in which the  $t$ -channel interaction is mediated by a graviton, the result is consistent with the result in eq. (50). However, this amplitude alone does not have enough structure to completely fix all of the unknown functions appearing in eq. (49). In fact, it is important in the previous calculations that we had the fully covariant string amplitudes without any restrictions on the polarization tensors in order to completely fix the graviton source. Finally we note that a more careful examination of the case  $p = 8$  shows that the pole is cancelled by momentum factors in the vertices for *all* of the amplitudes — see Appendix A.1. Hence our present analysis does not determine the sources for the eight-branes.

To determine the antisymmetric tensor source, we considered  $B$ - $\phi$  and  $B$ - $h$  scattering. In these cases though, the string scattering amplitude (11) is easily shown to vanish and so one concludes that the *linear* source term for  $B_{\mu\nu}$  in eq. (46) precisely vanishes.

$$\tilde{S}_B^{\mu\nu} = 0 \quad (51)$$

One can also verify these results for the sources of the dilaton and graviton fields, eqs. (47) and (50), by considering the scattering of the Ramond-Ramond tensor fields from the  $Dp$ -branes. The relevant terms in the low energy effective action are

$$I = \int d^{10}x \sqrt{-g} \sum_n \left( -\frac{8}{n!} F_{(n)} \cdot F_{(n)} e^{(5-n)\frac{\kappa}{\sqrt{2}}\phi} \right) . \quad (52)$$

For the type IIa superstring, the sum runs over  $n = 1, 3$  and  $5$ , while for the type IIb theory, the sum includes  $n = 2$  and  $4$ . An added complication is that  $F_{(5)}$  should be a self-dual field strength for which no covariant action exists. The above non-self-dual action will yield the correct type IIb equations of motion when the self-duality constraint is imposed by hand [23] — *i.e.*, one makes the substitution  $F_{(5)} \rightarrow F_{(5)} + *F_{(5)}$ . One can verify that this action (52) reproduces the three-point string amplitudes on the sphere for two R-R fields scattering with a graviton or dilaton. For a complete description of all of the  $NS^2$ - $R^2$ - $R^2$  amplitudes, one would have to include various Chern-Simon interactions between  $B_{\mu\nu}$  and the R-R fields, but we will not be interested in these since the Kalb-Ramond source vanishes, eq. (51).

The Feynman diagrams corresponding to the two-point scattering of R-R fields from the  $Dp$ -branes are in figure (2). The scattering amplitude is

$$A'_{FF} = i\tilde{S}_h^{\mu\nu} (\tilde{G}_h)_{\mu\nu,\lambda\rho}(k^2) (\tilde{V}_{hFF})^{\lambda\rho} + i\tilde{S}_\phi \tilde{G}_\phi(k^2) \tilde{V}_{\phi FF}$$

where

$$\begin{aligned} (\tilde{V}_{hFF})^{\lambda\rho} &= i\kappa \frac{16}{n!} \left[ 2n (F_{1(n)})^{(\lambda}{}_{\nu_2\nu_3\cdots\nu_n} (F_{2(n)})^{\rho)\nu_2\nu_3\cdots\nu_n} - \eta^{\lambda\rho} F_{1(n)} \cdot F_{2(n)} \right] \\ V_{\phi F(n)F(n)} &= -i\kappa \frac{8\sqrt{2}}{n!} (5-n) F_{1(n)} \cdot F_{2(n)} \end{aligned}$$

where we have left the external momenta and polarization tensors in the covariant form of a linearized field strength as in eq. (27). Substituting in our previous results for  $\tilde{S}_\phi$  and  $\tilde{S}_h^{\mu\nu}$  in eqs. (47) and (50),  $A'_{FF}$  reduces to

$$A'_{FF} = i\kappa T_p \frac{8}{n!} \frac{1}{k^2} \left( \text{Tr}(D) F_{1(n)} \cdot F_{2(n)} - 2n D^\lambda{}_\rho (F_1)_{\lambda\nu_2\nu_3\cdots\nu_n} (F_2)^{\rho\nu_2\nu_3\cdots\nu_n} \right) . \quad (53)$$

To demonstrate that this result agrees with the string amplitude, we focus on the  $a_1$  contribution in the kinematic factor (29)

$$\begin{aligned} a_1 &= -\frac{1}{2} \text{Tr}(P_- \Gamma_{1(n)} M \gamma_\mu C^{-1} M^T \Gamma_{2(m)}^T C \gamma^\mu) \\ &= -\frac{1}{2} \text{Tr} \left( P_- \Gamma_{1(n)} \gamma^\mu \Gamma_{2(m)} \gamma^\nu \right) D_{\mu\nu} (-1)^{\frac{1}{2}m(m+1)} \\ &= -\frac{8}{n!} \delta_{mn} \left[ \text{Tr}(D) F_{1(n)} \cdot F_{2(n)} - 2n D^\lambda{}_\kappa F_{1\lambda\nu_2\nu_3\cdots\nu_n} F_2^{\kappa\nu_2\nu_3\cdots\nu_n} \right] \end{aligned}$$

where we have applied various spinor matrix identities from Appendix B. Combining this result with eq. (44), we find that the result coincides precisely with eq. (53). As an aside, we also note that this explicit evaluation of  $a_1$  reproduces the result of Ref. [7]. The agreement between the field theory and string scattering of R-R particles was also found in the latter reference.

From the dilaton and graviton sources, it is a simple matter to calculate (the Fourier transform of) the corresponding long range fields around the  $Dp$ -branes. These fields

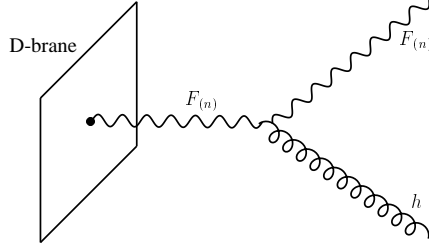


Figure 3: Feynman diagram for graviton–R-R tensor scattering from a D-brane

are precisely the product of the source and the Feynman propagator in the transverse momentum space. Hence the long range dilaton field is

$$\begin{aligned}\tilde{\phi}(k^2) &= i\tilde{S}_\phi \tilde{G}_\phi(k^2) = -\frac{T_p}{4\sqrt{2}} \frac{2 + \text{Tr}(D)}{k^2} \\ &= -\frac{T_p}{2\sqrt{2}} \frac{p-3}{k^2} .\end{aligned}\tag{54}$$

Similarly the long range gravitational field becomes

$$\begin{aligned}\tilde{h}_{\mu\nu}(k^2) &= i\tilde{S}_h^{\lambda\rho} (\tilde{G}_h)_{\lambda\rho,\mu\nu} \\ &= -\frac{T_p}{8k^2} ((7-p) V_{\mu\nu} - (p+1) N_{\mu\nu})\end{aligned}\tag{55}$$

Of course given eq. (51), the long range antisymmetric tensor field vanishes.

We would also like to determine the long range Ramond-Ramond fields around the  $Dp$ -brane. In this case, introducing a local source for a “magnetic” charge is a slight complication, which we avoid by working directly with the background fields. In this case, one expands the action (52) around a background field strength (which satisfies the equations of motion). The relevant amplitudes arise from three-point amplitudes involving one background field and two external particles. Essentially as in eqs. (54) and (55), one is replacing the source and propagator in the previous calculations by the background field in the amplitude. Here we consider a scattering process in which a graviton converts to a R-R tensor field. The Feynman diagram is shown in figure (3), and the corresponding scattering amplitude is

$$A'_{hF} = i \kappa n \frac{32}{n!} \varepsilon_2^{\lambda\mu} (F_{1(n)})_{\lambda}{}^{\nu_2\cdots\nu_n} (\tilde{F}_{(n)})_{\mu\nu_2\cdots\nu_n}$$

where  $\varepsilon_2$ ,  $F_{1(n)}$  and  $\tilde{F}_{(n)}$  are the graviton polarization, the external R-R particle’s (linearized) field strength, and the background R-R field strength, respectively. Note that the interaction requires that the form degree  $n$  is the same for both the external and the background R-R fields. The massless  $t$ -channel pole for the string scattering amplitude

$A_{hF}$  comes from  $a_1$  in eq. (30) which yields

$$a_1 = \pm i \frac{8\sqrt{2}}{(n-1)!} \varepsilon_2^{\lambda\mu} (F_{1(n)})_{\lambda}{}^{\nu_2 \dots \nu_n} \times \begin{cases} nk_{[\nu_n} (\epsilon^v)_{\mu\nu_2 \dots \nu_{n-1}] } & \text{for } p = n - 2 \\ k^\rho (\epsilon^n)_{\rho\mu\nu_2 \dots \nu_n} & \text{for } p = 8 - n \end{cases} .$$

where the  $\pm$  sign is the same as that appearing in choice made for  $M$  from eq. (68). Comparing the string amplitude  $A_{hF}$  with  $A'_{hF}$ , one obtains

$$\tilde{F}_{\nu_1\nu_2 \dots \nu_n} = \mp i \frac{T_p}{2\sqrt{2}k^2} \times \begin{cases} nk_{[\nu_n} (\epsilon^v)_{\nu_1 \dots \nu_{n-1}] } & \text{for } p = n - 2 \\ k^\mu (\epsilon^n)_{\mu\nu_1\nu_2 \dots \nu_n} & \text{for } p = 8 - n \end{cases} .$$

Anticipating that the first line corresponds to an “electric” field, we see Dp-branes with  $p = -1, 0, 1, 2$ , and 3 have “electric” fields with  $n = 1, 2, 3, 4$  and 5, respectively. While the second line above shows that Dp-branes with  $p = 3, 4, 5, 6$  and 7 have “magnetic” fields with  $n = 5, 4, 3, 2$  and 1, respectively. As expected the D3-brane simultaneously carries electric and magnetic fields from the self-dual five-form.

### 4.3 Comparison of fields

To make a precise comparison of the results in the previous two sections, we must first take into account that the low energy string actions, (45) and (52), have a different normalization from that used to derive the  $p$ -brane solutions, eq. (33). Hence we make use the following field redefinitions for the fields appearing in section 4.1

$$\begin{aligned} \hat{h}_{\mu\nu} &\equiv 2\kappa h'_{\mu\nu} \\ \sqrt{\hat{\gamma}} \hat{\phi} &\equiv \sqrt{2}\kappa \phi' \\ \hat{F}_{(n)} &\equiv 4\sqrt{2}\kappa F'_{(n)} \end{aligned} \tag{56}$$

and we identify the constants  $\sqrt{\hat{\gamma}} \hat{a} \equiv a'$  and  $8\pi \hat{G}_N = \kappa^2$ , as well as setting  $d = 10$ . With these choices the  $p$ -brane action (33) becomes

$$\hat{I} = \int d^{10}x \sqrt{-g'} \left[ \frac{1}{2\kappa^2} R'(g') - \frac{1}{2} (\nabla \phi')^2 - \frac{8}{n!} e^{-a' \sqrt{2}\kappa \phi'} F'^2_{(n)} \right] .$$

where  $g'_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h'_{\mu\nu}$ . Finally this action may be identified with the relevant part of the effective superstring action by setting  $a' = (n-5)/2$ .

To compare the low energy  $p$ -brane solutions to the long-range Dp-brane fields, we make a Fourier transform of the asymptotic fields of sect. 4.1. The essential transform is that for  $G(|\vec{x}|/\ell)$  in eq. (37) which is the Green’s function in the transverse space.

$$\tilde{G}(\vec{k}^2) = \frac{\mathcal{A}_{\bar{p}-1} \ell^{\bar{p}-2}}{\vec{k}^2} \tag{57}$$

where  $\vec{k}$  is a wave-vector in the subspace orthogonal to the  $p$ -brane, and as before  $\mathcal{A}_{\bar{p}-1}$  is the area of a unit  $(\bar{p}-1)$ -sphere. The Green’s function (57) has the same form as the

Feynman propagators appearing in the field theory calculations with  $k^2 = \vec{k}^2$  — recall that the  $t$ -channel momentum vector is a spatial vector in the transverse subspace (see Appendix A).

Now in terms of the primed fields introduced in eq. (56), the Fourier transform of the asymptotic fields (38–39) of the  $p$ -brane solutions with “electric” charge are

$$\begin{aligned}\tilde{h}'_{\mu\nu} &\simeq \frac{\mathcal{A}_{\bar{p}-1}\mu\ell^{\bar{p}-2}}{\kappa} \frac{1}{\vec{k}^2} \text{diag}(-\alpha, \alpha, \dots, \alpha, \beta, \dots, \beta) \\ \tilde{\phi}' &\simeq -\frac{\hat{a}\sigma\mathcal{A}_{\bar{p}-1}\mu\ell^{\bar{p}-2}}{2\sqrt{2}\kappa} \frac{1}{\vec{k}^2} \\ \tilde{F}'_{(n)} &\simeq \mp i \frac{\sqrt{\sigma}\mathcal{A}_{\bar{p}-1}\mu\ell^{\bar{p}-2}}{4\kappa} \frac{1}{\vec{k}^2} \vec{k} \cdot dx \wedge (\epsilon^v) \quad .\end{aligned}$$

In this case with  $d = 10$ ,  $\bar{p} = 9 - p$  and  $p = n - 2 \leq 3$ , the constants (36) reduce to

$$\begin{aligned}\alpha &= -\frac{7-p}{16} & \beta &= \frac{p+1}{16} \\ \sigma &= \frac{1}{2} & \hat{a} &= n-5 = p-3 \quad .\end{aligned}\tag{58}$$

Hence we have complete agreement between the long-range  $Dp$ -brane fields and those above if  $\kappa T_p = \mathcal{A}_{8-p}\mu\ell^{7-p}/2$ . Further applying the mass formula (41), we find the mass per unit  $p$ -volume is given by  $M_p = T_p/\kappa$ , while the electric charge per unit  $p$ -volume (42) is  $Q_p = \pm\sqrt{2}T_p$ .

For the magnetically charged  $p$ -branes, the metric is unchanged while the Fourier transform of eq. (40) yields

$$\begin{aligned}\tilde{\phi}' &\simeq +\frac{\hat{a}\sigma\mathcal{A}_{\bar{p}-1}\mu\ell^{\bar{p}-2}}{2\sqrt{2}\kappa} \frac{1}{\vec{k}^2} \\ \tilde{F}'_{(n)} &\simeq \mp i \frac{\sqrt{\sigma}\mathcal{A}_{\bar{p}-1}\mu\ell^{\bar{p}-2}}{4\kappa} \frac{1}{\vec{k}^2} i_{\vec{k}}(\epsilon^n) \quad .\end{aligned}$$

Again with  $d = 10$ ,  $\bar{p} = 9 - p$  and now  $p = 8 - n \geq 3$ , one finds the same expressions for the constants  $\alpha$ ,  $\beta$  and  $\sigma$  as in (58), while

$$\hat{a} = n - 5 = 3 - p$$

has the opposite sign. Again with  $\kappa T_p = \mathcal{A}_{8-p}\mu\ell^{7-p}/2$ , one finds precise agreement between the fields of the  $Dp$ -branes and extremal  $p$ -brane solutions. The magnetic charge per unit  $p$ -volume (43) is  $Q_p = \pm\sqrt{2}T_p$ .

## 5 Discussion

In this paper, we have presented detailed calculations of all two-point amplitudes describing massless closed type II superstrings scattering from a Dirichlet  $p$ -brane in ten dimensions.

Using these results we derived the long range fields around  $Dp$ -branes for  $0 \leq p \leq 7$ , and found that they correspond to those of the low energy solutions describing a supersymmetric  $p$ -brane carrying a R-R charge.

One of the most interesting aspects of this work was the observation that these closed string two-point amplitudes describing scattering from a  $Dp$ -brane are simply related to four-point amplitudes for type I open superstrings. This relation allowed the D-brane amplitudes to be easily determined using the previously calculated open string amplitudes. This result is similar to the work of Kawai, Lewellen and Tye [12], who were able to express tree-level closed string amplitudes as products of pairs of open string amplitudes along with certain “sewing” factors. This structure arises because of the independence of the correlation functions for the left- and right-moving sectors. The present situation is similar except that the D-brane boundary naturally “sews” the right- and left-movers together in a single open string amplitude. In Ref. [12], the results for tree-level string scattering are extended to amplitudes with an arbitrary number of closed string states. It would be interesting to find a similar extension relating a D-brane scattering amplitude for  $N$  closed strings to a Type I amplitude for  $2N$  open strings.

Our normalizations are chosen such that  $T_p$  coincides precisely with the string tension in a standard analysis, *e.g.*, [5]. The physical tension [5] is precisely the mass per unit  $p$ -volume  $\tau_p = T_p/\kappa = M_p$ . For a fundamental  $Dp$ -brane then,  $T_p = \sqrt{\pi}/(2\pi\sqrt{\alpha'})^{p-3}$ . Higher multiples of this fundamental value of  $T_p$  would arise for  $Dp$ -branes which are superpositions or bound states of fundamental branes. Similarly our charges correspond precisely to those given in the standard analysis [3, 5], *i.e.*,  $\mu_p = Q_p$  with  $Q_p^2 = 2T_p^2$ . The sign of the charges is undetermined. It is fixed in the conformal field theory calculations by making a choice of sign for the matrix  $M$  appearing in the spin operators — see Appendix B. It is remarkable that the conformal field theory allows just enough ambiguity in  $M$  to accommodate both branes and anti-branes.

This matrix also plays an important role in the spacetime supersymmetry of the D-brane amplitudes, as follows. On a closed type II superstring world sheet, one can construct two independent spacetime supersymmetry currents [14, 9]

$$Q_L = \epsilon_L^A e^{-\phi(z)/2} S_A(z) \quad \text{and} \quad Q_R = \epsilon_R^A e^{-\tilde{\phi}(\bar{z})/2} \tilde{S}_A(\bar{z})$$

with independent actions on the left- and right-moving modes on the world sheet. Hence  $\epsilon_L$  and  $\epsilon_R$  are independent Majorana-Weyl spacetime supersymmetry parameters in the Type II theory. In the Type I open superstring theory, the left- and right-movers are tied together by the world sheet boundaries, and hence there is a single SUSY current to act on the open string states. Given the relation of open string and D-brane scattering amplitudes, the latter would inherit this single spacetime supersymmetry of the open superstring amplitudes. Hence there will be a single SUSY current which acts consistently on both the left- and right-moving components of the closed string vertex operators. If we extend the spin fields in the SUSY currents to the entire complex plane using eq. (25) then the right-moving current becomes

$$Q_R = \epsilon_R^B M_B^A e^{-\phi(\bar{z})/2} S_A(\bar{z}) \quad .$$

In order to construct the single consistent current then we must set

$$\epsilon_L^A = \epsilon_R^B M_B^A \quad .$$

This then is a conformal field theory approach to understanding that the D-branes preserve precisely one half of the spacetime supersymmetries of the Type II theory[3].

The long range fields around the  $Dp$ -branes were determined in sect. 4 by considering the massless  $t$ -channel poles in the string scattering amplitudes. As well as these poles, the string amplitudes contain an infinite number of massive poles as well. Using a standard expansion of the Euler beta function (see *e.g.*, [24]), one finds that eq. (21) may be written as

$$A = -i \kappa T_p \left( \frac{a_1}{k^2} - \frac{a_2 + 2q^2 a_1}{k^2 + 2} + \frac{(2q^2 - 1)(a_2 + 2q^2 a_1)}{k^2 + 4} + \dots \right) \quad (59)$$

where  $k^2 = -t$ . Each of the higher  $t$ -channel poles represents a massive closed string state coupling to the  $Dp$ -brane. From this point of view, the  $Dp$ -brane provides a  $\delta$ -function source for each of these massive states, just as it does for the massless states. The same set of  $\delta$ -functions in the transverse coordinates appears in the construction of a boundary state description of a D-brane (see *e.g.*, [25]). This would then lend itself to an interpretation of D-branes as objects of zero thickness. However, since as seen above a D-brane is not only a source of the massless fields but also massive fields with  $m^2 = 4n/\alpha'$ , the conventional (low energy) spacetime picture will breakdown at distances of the order of  $\sqrt{\alpha'}$ . It is within this range that the full closed string spectrum makes its presence felt. Hence from this perspective, one would ascribe a thickness of the order of  $\sqrt{\alpha'}$  to D-branes.

Of course, the preceding discussion ignores the stringy nature of the amplitudes and in particular their  $s$ - $t$  channel duality. As emphasized by [6], the amplitudes can be reorganized as a series of  $s$ -channel (or  $q^2$ ) poles

$$A = i \frac{\kappa T_p}{2} \left( \frac{a_2}{2q^2} + \frac{a_1 + k^2 a_2/2}{2q^2 + 1} + \frac{(1 - k^2/2)(a_1 + k^2 a_2/2)}{2q^2 + 2} + \dots \right) \quad . \quad (60)$$

In this case, the poles coincide with open string states moving along the D-brane. These in turn correspond to excitations of internal modes or deformations of the D-brane. Hence this point of view naturally displays a smearing of the D-brane again on the order of  $\sqrt{\alpha'}$  [6].

An exception to the above form of the amplitudes is the special case of  $p = -1$ . The  $D(-1)$ -brane corresponds to a Euclidean instanton which then has no world volume and hence there is no momentum flow parallel to the world volume — *i.e.*,  $q^\mu = 0$ . As noted in [6], the amplitudes can then not take the form in eq. (60) and further without  $s$ - $t$  channel duality one would expect that the infinite series in the  $t$ -channel expression (59) is also truncated. Explicitly calculating the amplitudes considered in the present investigation (*i.e.*, for massless external states) shows that in fact only the massless  $t$ -channel pole survives. This was already observed in Ref. [6] in the scattering amplitude



for two antisymmetric tensors

$$A_{BB} = i\kappa T_p \left( \frac{4 p_1 \cdot \varepsilon_2 \cdot \varepsilon_1 \cdot p_2}{t} - \text{Tr}(\varepsilon_1 \cdot \varepsilon_2) \right) . \quad (61)$$

(Also the graviton-graviton amplitude vanishes completely [6].) A similar result holds for graviton-dilaton scattering and graviton-R-R tensor scattering

$$\begin{aligned} A_{h\phi} &= -i2\sqrt{2}\kappa T_p \frac{p_2 \cdot \varepsilon_1 \cdot p_2}{t} \\ A_{hF} &= -8\sqrt{2}\kappa T_p \frac{\varepsilon_1^{\lambda\mu} (F_2)_{\lambda k_\mu}}{t} . \end{aligned} \quad (62)$$

These amplitudes were determined by substituting  $D_{\mu\nu} = -\eta_{\mu\nu}$  into our general results and taking the limit that  $q^2 \rightarrow 0$ . Recalculating these amplitudes directly from conformal field theory reproduces the same answers. Given that these three massless poles are the same as in our general formulae, the analysis of sect. 4 applies to D(-1)-branes as well. Hence their long range fields are those of the low energy solutions discussed in Ref. [17] (see also [7]).

While no massive poles in these particular amplitudes (61-62), one should not conclude that no massive string states couple to the D(-1)-branes. We expect that such poles and couplings would make their appearance in scattering amplitudes involving massive external strings. This should be evident given the complex form of the boundary state describing a D(-1)-brane [26]. Hence even for these instantons, one would ascribe a thickness of the order of  $\sqrt{\alpha'}$ .

Our analysis of the long range fields is not valid in the case of domain walls (for which  $p = 8$ ) – see Appendix A.1. Naively the amplitudes for two NS-NS or two R-R states appear to give the expected long range gravitational and dilaton fields. However, a closer inspection of these amplitudes shows that because of the unusual kinematics of this configuration, there is no pole at  $t = 0$ . This result is in fact in agreement with the analogous field theory calculations. In order to overcome this difficulty, one would have to begin by extending the source action (46) to include nonlinear terms. These interactions are required to understand the two-particle contact terms that arise in the string amplitudes. It should be possible to extract the necessary terms from the covariant Born-Infeld actions constructed as the effective D-brane actions [20, 21]. Another necessary ingredient here would be a better understanding of the vertex operator description of the  $n = 10$  R-R field strength whose charge is carried by the D8-branes.

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## A Kinematics of Two-Point Amplitudes

In the calculation of Dirichlet  $p$ -brane scattering amplitudes, it is convenient to introduce two sets of frame fields:  $v^a_\mu$  with  $a = 0, \dots, p$ , and  $n^a_\mu$  with  $a = p+1, \dots, 9$ . So the  $v^a$  are unit vectors tangent to the  $p$ -brane's world-volume while the  $n^a$  are orthogonal to the world-volume. The vector (Greek) indices are raised and lowered as usual with the metric  $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$ , and similarly the frame (Latin) indices are raised and lowered with  $\eta_{ab}$ . Then one has

$$n^a \cdot n^b = \eta^{ab} \quad v^a \cdot v^b = \eta^{ab} \quad n^a \cdot v^b = 0 .$$

Recall that it is the coordinate fields running orthogonal to the  $Dp$ -brane and hence parallel to the  $n^a_\mu$ , which satisfy the Dirichlet boundary conditions.

Given this separation of the frame fields, one can construct two useful projection operators, namely,

$$N_{\mu\nu} = n^a_\mu n_{a\nu} \quad \text{and} \quad V_{\mu\nu} = v^a_\mu v_{a\nu}$$

where  $N$  projects vectors into the transverse subspace or the subspace orthogonal to the  $Dp$ -brane, and  $V$  projects vectors into the subspace parallel to the  $Dp$ -brane. Hence  $N_\mu^\lambda N_{\lambda\nu} = N_{\mu\nu}$ ,  $V_\mu^\lambda V_{\lambda\nu} = V_{\mu\nu}$  and  $N_\mu^\lambda V_{\lambda\nu} = 0$ . Completeness of the basis of frames also yields

$$\eta_{\mu\nu} = V_{\mu\nu} + N_{\mu\nu} .$$

The matrix appearing in the correlation functions for coordinate fields (6) and world-sheet spinors (7) is given by

$$D_{\mu\nu} = V_{\mu\nu} - N_{\mu\nu} . \tag{63}$$

Combining the above expressions, one also has  $D_{\mu\nu} = \eta_{\mu\nu} - 2N_{\mu\nu} = 2V_{\mu\nu} - \eta_{\mu\nu}$ . Also note that  $D_\mu^\lambda D_{\lambda\nu} = \eta_{\mu\nu}$ .

These frames are also useful in defining certain volume forms. In the subspace parallel to the brane's world-volume, one has a  $(p+1)$ -form

$$(\epsilon^v)_{\mu_0 \dots \mu_p} = (p+1)! v^0_{[\mu_0} \dots v^p_{\mu_p]} ,$$

while in the normal subspace one can define the  $(9-p)$ -form

$$(\epsilon^n)_{\mu_{p+1} \dots \mu_9} = (9-p)! n^{p+1}_{[\mu_{p+1}} \dots n^9_{\mu_9]} .$$

The antisymmetrization of the indices above is normalized such that both  $\epsilon^v$  and  $\epsilon^n$  take values  $\pm 1$  and 0. These two forms are related by the identities

$$\begin{aligned} (\epsilon^v)_{\mu_0 \dots \mu_p} &= \frac{1}{(9-p)!} \epsilon_{\mu_0 \dots \mu_p \mu_{p+1} \dots \mu_9} (\epsilon^n)^{\mu_{p+1} \dots \mu_9} \\ (\epsilon^n)_{\mu_{p+1} \dots \mu_9} &= -\frac{1}{(p+1)!} (\epsilon^v)^{\mu_0 \dots \mu_p} \epsilon_{\mu_0 \dots \mu_p \mu_{p+1} \dots \mu_9} . \end{aligned}$$

where  $\epsilon$  is the regular volume form in the full ten-dimensional Minkowski space. As forms, one also has  $\epsilon = (\epsilon^v) \wedge (\epsilon^n)$ .

To describe the kinematics of the string scattering amplitudes in a D-brane background, we divide the momentum vectors into their parallel and transverse components, *i.e.*,  $p^\mu = (N \cdot p)^\mu + (V \cdot p)^\mu$ . Now one has only momentum conservation in the parallel subspace, *i.e.*,  $\sum_i (V \cdot p_i)^\mu = 0$ , so for the two point amplitudes considered in the paper, we may define

$$(V \cdot p_1)^\mu = q^\mu = -(V \cdot p_2)^\mu .$$

Further since we are considering massless external states, one finds

$$(N \cdot p_1)^2 = -q^2 = (N \cdot p_2)^2 . \quad (64)$$

So  $q^\mu$ , the momentum flowing through the scattering amplitude parallel to the  $p$ -brane, must be time-like since by definition  $(N \cdot p)^\mu$  is space-like.

As well as  $p_1$  and  $p_2$ ,  $D \cdot p_1$  and  $D \cdot p_2$  appear as momenta appear in the vertex operators in sects. 2 and 3. Some useful identities satisfied by these vectors are

$$\begin{aligned} (p_1 + D \cdot p_1 + p_2 + D \cdot p_2)^\mu &= 0 \\ p_1 \cdot D \cdot D \cdot p_1 &= p_1^2 = 0 \\ p_2 \cdot D \cdot D \cdot p_2 &= p_2^2 = 0 \\ p_1 \cdot D \cdot p_1 &= p_2 \cdot D \cdot p_2 = 2q^2 \\ p_1 \cdot D \cdot D \cdot p_2 &= p_1 \cdot p_2 = -t/2 \\ p_1 \cdot D \cdot p_2 &= -2q^2 + t/2 \end{aligned}$$

where  $t = -(p_1 + p_2)^2$  is the momentum transfer to the  $Dp$ -brane.

## A.1 Domain Wall Kinematics

The kinematics of our two-point amplitudes is special for the case of  $p = 8$ . In this case, the  $p$ -brane forms a domain wall dividing the ten-dimensional spacetime into two halves. The transverse space is only one-dimensional running along  $x^9$ , and so it is only the ninth component of the momentum vectors which is not conserved. Hence eq. (64) becomes

$$(p_1^9)^2 = -q^2 = (p_2^9)^2 .$$

Since there is a single component of momentum appearing here, one has either  $p_1^9 = -p_2^9$  or  $p_1^9 = p_2^9$ . In the first case, there is no momentum transfer to the brane, *i.e.*, the momentum is precisely conserved, and hence there is no string scattering. We will not consider this case further. In second case, one has

$$t = -(p_1 + p_2)^2 = -(p_1^9 + p_2^9)^2 = -4(p_1^9)^2 = 4q^2 ,$$

and hence the momentum transfer is directly proportional to the momentum flowing parallel to the brane. This result allows one to rewrite the universal form of the amplitudes (21) as

$$\begin{aligned} A(1, 2) &= -i \frac{\kappa T_p}{2} \frac{\Gamma(-2q^2)\Gamma(2q^2)}{\Gamma(1)} K(1, 2) \\ &= i \frac{\kappa T_p}{4q^2} \frac{\pi}{\sin(2\pi q^2)} K(1, 2) . \end{aligned}$$

This expression combines all of the poles in the single sine function factor. One must remember though that the poles for  $q^2 < 0$  are associated with the  $s$ -channel, while those with  $q^2 > 0$  are  $t$ -channel poles. Naively there is a problem at  $q^2 = 0$  where one might expect a pole in both channels. In fact, the apparent double pole can be seen to be reduced to a simple pole by the fact that all of the terms in any of the kinematic factors  $K$  always contain factors of  $q^2$  or  $t = 4q^2$ . However, a closer examination shows that in fact there is no pole at all. For example consider the factors appearing in  $K_{NS-NS, NS-NS}$  in eqs. (12) and (13). It is straightforward to show that all of the nonvanishing inner products of momenta with polarization tensors also yield factors of  $p_1^9$ , *e.g.*,

$$\begin{aligned} \varepsilon_{1\mu\nu} p_2^\nu &= \varepsilon_{1\mu\nu} V^{\nu\rho} p_{2\rho} + \varepsilon_{1\mu\nu} N^{\nu\rho} p_{2\rho} \\ &= -\varepsilon_{1\mu\nu} V^{\nu\rho} p_{1\rho} + \varepsilon_{1\mu\nu} N^{\nu\rho} p_{1\rho} \\ &= -\varepsilon_{1\mu\nu} p_1^\nu + 2\varepsilon_{1\mu\nu} N^{\nu\rho} p_{1\rho} \\ &= 2\varepsilon_{1\mu\nu} N^{\nu\rho} p_{1\rho} = 2\varepsilon_{1\mu 9} p_1^9 , \end{aligned}$$

where  $N^{\nu\rho} p_{1\rho} = \delta_9^\nu p_1^9$ . Similarly many of the terms also vanish through identities such as  $\varepsilon_1 \cdot D \cdot p_2 = 0$ . In any event, all of the remaining terms carry a factor of  $(p_1^9)^2 = -q^2$  which cancels even the remaining pole.

The absence of a massless  $t$ -channel pole is the reason that we make no detailed comparison of the string and field theory scattering for eight-branes in sect. 4. This unusual kinematics applies equally well for the field theory calculations as the string amplitudes. In both cases then, these  $t$ -channel terms make contributions equivalent to two-particle contact terms on the  $Dp$ -brane. In the string amplitude, one also expects to find such contributions from explicit contact interactions and also from the remnants of the massless  $s$ -channel pole. In fact there are a large number of cancellations amongst these terms in the string amplitudes. For example, the  $NS^2$ - $NS^2$  amplitude in eq. (11) reduces to

$$A = i \frac{\kappa T_p}{2} \text{Tr}(\varepsilon_1 \cdot D \cdot \varepsilon_2 \cdot D) + O(q^2) \quad (65)$$

for small  $q^2$ . The appearance of two  $D_{\mu\nu}$  in this term suggests that its origin is in an explicit contact interaction as illustrated in figure (4). In the field theory, one expects that the covariant extension of the source actions [20, 21] would also naturally introduce multiparticle contact terms to our source actions (46). Hence our present field theory

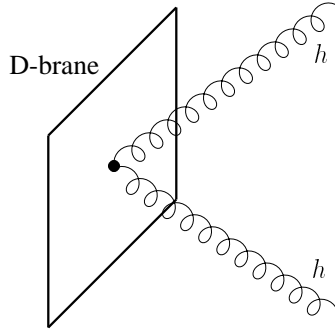


Figure 4: Feynman diagram for a two-graviton contact interaction on a D-brane

calculations can not be expected to account for a term such as that in eq. (65). Introducing a fully covariant source action would be one step towards overcoming these obstructions to determining the eightbrane background fields, but one would still have to unravel the  $s$ -channel contributions in the string amplitude.

## B Spinors and Dirac Matrices

We have adapted our spinor conventions within conformal field theory from Refs. [9, 10]. We distinguish spinor and adjoint spinor indices as (upper case Latin) subscripts and superscripts, respectively [9]. The charge conjugation matrix acts as a metric for raising or lowering the spinor indices, *e.g.*,

$$u^A = C^{AB} u_B \quad u_A = C_{AB}^{-1} u^B \quad . \quad (66)$$

Of course, we also have  $C_{AB}^{-1} C^{BC} = \delta_A^C$ . Following [9], we adopt the convention that  $C^{AB} = -C^{BA}$ . The spinors appearing in the open string amplitudes (17-20) are Majorana, and hence in our notation,  $\bar{u}^A = u_B C^{BA} = -u^A$ .

For our Dirac matrix conventions, we begin with the anticommutator

$$\{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu} \quad .$$

Unless otherwise indicated, implicitly in the above expression and the various products of sect. 3, we understand that the indices on the Dirac matrices appear as  $(\gamma^\mu)_A^B$ . Using eq. (66), we write

$$(\gamma^\mu)_{AB} = C_{BC}^{-1} (\gamma^\mu)_A^C \quad (\gamma^\mu)^{AB} = C^{AC} (\gamma^\mu)_C^B \quad .$$

Then we chose a representation of the Dirac matrices in which [9]

$$(\gamma^\mu)_{AB} = (\gamma^\mu)_{BA} \quad .$$

In our notation, we write the transpose matrices as

$$(\gamma_\mu^T)^B{}_A = (\gamma_\mu)_A{}^B$$

and with the above conventions, we have

$$C^{AC} (\gamma_\mu)_C{}^D C_{DB}^{-1} = -(\gamma_\mu^T)^A{}_B \quad \text{or} \quad (\gamma_\mu)_A{}^B = -C_{AC}^{-1} (\gamma_\mu^T)^C{}_D C^{DB} .$$

It is also useful to note the general result that  $R_{AB} S^{BC} = -R_A{}^B S_B{}^C$  where  $R$  and  $S$  are arbitrary products of  $\gamma$ -matrices.

We also have

$$\gamma_{11} = \frac{1}{10!} \epsilon_{\mu_0 \dots \mu_9} \gamma^{\mu_0} \dots \gamma^{\mu_9} = \gamma^0 \gamma^1 \dots \gamma^9$$

which satisfies

$$(\gamma_{11})^2 = 1 , \quad (\gamma_{11})_{AB} = (\gamma_{11})_{BA} , \quad \gamma_{11}^T = -C \gamma_{11} C^{-1} .$$

With  $\gamma_{11}$  one constructs the chiral projection operators,  $P_\pm = \frac{1}{2}(1 \pm \gamma_{11})$ . Two useful identities are

$$(P_\pm)_{AB} = -(P_\mp)_{BA} \quad P_\pm^T = C P_\mp C^{-1} .$$

In constructing the R-R vertex operators, one also encounters the matrices  $\Gamma_{(n)}$  which are defined in eq. (26) as

$$\Gamma_{(n)} = \frac{a_n}{n!} F_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n} .$$

These can be shown to satisfy

$$(\Gamma_{(n)})_{AB} = -(-1)^{\frac{1}{2}n(n+1)} (\Gamma_{(n)})_{BA} \quad \Gamma_{(n)}^T = (-1)^{\frac{1}{2}n(n+1)} C \Gamma_{(n)} C^{-1} .$$

Given the above, one can show that  $P_- \Gamma_{(5)} = 0$  if the corresponding field strength is antiselfdual, *i.e.*,  $F_{(5)} = - * F_{(5)}$ , while  $P_- \Gamma_{(5)} = \Gamma_{(5)}$  for a selfdual field strength, *i.e.*,  $F_{(5)} = * F_{(5)}$ .

Finally, we would like to fix the matrix  $M$  required in eq. (25)

$$\tilde{S}_A(\bar{z}) \rightarrow M_A{}^B S_B(\bar{z}) \tag{67}$$

to extend the spin fields to the entire complex plane. We begin by considering the the following operator products [10]

$$\begin{aligned} \psi^\mu(z) S_A(w) &\sim (z-w)^{-\frac{1}{2}} \frac{1}{\sqrt{2}} (\gamma^\mu)_A{}^B S_B(w) + \dots \\ \tilde{\psi}^\mu(\bar{z}) \tilde{S}_A(\bar{w}) &\sim (\bar{z}-\bar{w})^{-\frac{1}{2}} \frac{1}{\sqrt{2}} (\gamma^\mu)_A{}^B \tilde{S}_B(\bar{w}) + \dots \end{aligned}$$

Making the replacement in eq. (67) as well as  $\tilde{\psi}^\mu \rightarrow D^\mu{}_\nu \psi^\nu$  in the second OPE, consistency with the first OPE requires that

$$(\gamma^\mu)_A{}^B = D^\mu{}_\nu (M^{-1} \gamma^\nu M)_A{}^B .$$

Alternatively, this identity may be written as  $(M \gamma^\mu) = D^\mu{}_\nu (\gamma^\nu M)$ . In other words,  $M$  (anti)commutes the  $\gamma^\mu$  when  $\mu$  corresponds to a direction orthogonal (parallel) to the Dp-brane world-volume. Hence one concludes

$$M = \begin{cases} a \gamma^0 \cdots \gamma^p & \text{for } p+1 \text{ odd} \\ b \gamma^0 \cdots \gamma^p \gamma_{11} & \text{for } p+1 \text{ even} \end{cases}$$

where  $a$  and  $b$  are undetermined phase factors. Note that in the scattering amplitudes the factor of  $\gamma_{11}$  for  $p+1$  even simply provides an extra sign in the amplitudes, and so this form essentially agrees with that in Ref. [7]. The phase factors  $a$  and  $b$  may be further fixed by the following OPE's [9]

$$\begin{aligned} S_A(z) S_B(w) &\sim (z-w)^{-\frac{5}{4}} C_{AB}^{-1} + \cdots \\ \tilde{S}_A(\bar{z}) \tilde{S}_B(\bar{w}) &\sim (\bar{z}-\bar{w})^{-\frac{5}{4}} C_{AB}^{-1} + \cdots \end{aligned}$$

In this case consistency with eq. (67) requires that

$$M_A{}^C M_B{}^D C_{CD}^{-1} = C_{AB}^{-1}.$$

Alternatively writing  $M C^{-1} M^T C = 1$ , we see that  $M^{-1} = C^{-1} M^T C$ . This relation fixes the phase factors to be  $a = \pm i$  and  $b = \pm 1$ . Hence one may write

$$M = \begin{cases} \frac{\pm i}{(p+1)!} (\epsilon^v)_{\mu_0 \cdots \mu_p} \gamma^{\mu_0} \cdots \gamma^{\mu_p} & \text{for } p+1 \text{ odd} \\ \frac{\pm 1}{(p+1)!} (\epsilon^v)_{\mu_0 \cdots \mu_p} \gamma^{\mu_0} \cdots \gamma^{\mu_p} \gamma_{11} & \text{for } p+1 \text{ even} \end{cases} \quad (68)$$

The remaining ambiguity in the sign ultimately determines the sign of the R-R charge carried by the Dp-brane, as shown in sect. 4. With our choice of conventions,  $M$  has the following symmetries

$$(M)_{AB} = (-1)^{\frac{1}{2}p(p+1)} (M)_{BA} \quad M^T = -(-1)^{\frac{1}{2}p(p+1)} C M C^{-1}.$$

## References

- [1] C.M. Hull and P.K. Townsend, Nucl. Phys. **B438** (1995) 109 [hep-th/9410167]; E. Witten, Nucl. Phys. **B443** (1995) 85 [hep-th/9503124]; P. Hořava and E. Witten, Nucl. Phys. **B460** (1996) 506 [hep-th/9510209]; J. Polchinski and E. Witten, Nucl. Phys. **B460** (1996) 525 [hep-th/9510169]; E. Witten, Nucl. Phys. **B460** (1996) 541 [hep-th/9511030]; K. Dasgupta and S. Mukhi, “Orbifolds of M Theory,” preprint hep-th/9512196; E. Witten, “Five-branes and M Theory on an Orbifold,” preprint hep-th/9512219; M.J. Duff, R. Minasian and E. Witten, “Evidence for Heterotic/Heterotic Duality,” preprint hep-th/9601036; N. Seiberg and E. Witten, “Comments on String Dynamics in Six-dimensions,” preprint hep-th/9603003. bibitem

- [2] J.H. Schwarz, “The Power of M Theory” preprint hep-th/9510086; C.M. Hull, “String Dynamics at Strong Coupling,” preprint hep-th/9512181; C. Vafa, “Evidence for F-Theory,” preprint hep-th/9602022; D. Kutasov and E. Martinec, “New Principles for String/Membrane Unification,” preprint hep-th/9602049; P.K. Townsend, “D-branes from M-branes,” preprint hep-th/9512062.
- [3] J. Polchinski, “Dirichlet-Branes and Ramond-Ramond Charges”, preprint hep-th/9510017.
- [4] J. Dai, R.G. Leigh and J. Polchinski, Mod. Phys. Lett. **A4** (1989) 2073; J. Polchinski, Phys. Rev. **D50** (1994) 6041.
- [5] J. Polchinski, S. Chaudhuri and C.V. Johnson, “Notes on D-Branes,” preprint hep-th/9602052.
- [6] I.R. Klebanov and L. Thorlacius, “The Size of  $p$ -Branes,” preprint hep-th/9510200.
- [7] S.S. Gubser, A. Hashimoto, I.R. Klebanov and J.M. Maldacena, “Gravitational lensing by  $p$ -branes,” preprint hep-th/9601057.
- [8] J.L.F. Barbon, “D-Brane Form-Factors at High Energy,” preprint hep-th/960198.
- [9] D. Friedan, “Notes on String Theory and Two Dimensional Conformal Field Theory,” in *Unified String Theories*, Eds. M.B. Green and D.J. Gross (World Scientific Publishing, 1986).
- [10] M.E. Peskin, “Introduction to String and Superstring Theory II,” in *From the Planck Scale to the Weak Scale: Toward a Theory of the Universe*, Proceedings of TASI ’86, Ed. H. Haber (World Scientific Publishing, 1987); V.A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, Nucl. Phys. **B288** (1987) 173.
- [11] J.H. Schwarz, Physics Reports **89** (1982) 223.
- [12] H. Kawai, D.C. Lewellen and S.-H.H. Tye, Nucl. Phys. **B269** (1986) 1.
- [13] M.B. Green and M. Gutperle, “Comments on Three-Branes,” preprint hep-th/9602077.
- [14] D. Friedan, S. Shenker and E. Martinec, Phys. Lett. **160B** (1985) 55; Nucl. Phys. **B271** (1986) 93.
- [15] H. Lu and C.N. Pope, “An Approach to the Classification of P-Brane Solitons,” preprint hep-th/9601089; “P-Brane Solitons in Maximal Supergravities,” preprint hep-th/9512012; H. Lu, C.N. Pope, E. Sezgin and K.S. Stelle, “Stainless Super P-Branes,” preprint hep-th/9508042; E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, “Duality of Type II 7 Branes and 8 Branes,” preprint hep-th/9601150; R.R. Khuri and R.C. Myers, “Rusty Scatter Branes,” preprint hep-th/9512061.



- [16] M.J. Duff, R.R. Khuri and J.X. Lu, Phys. Rep. **259** (1995) 213.
- [17] G.W. Gibbons, M.B. Green and M.J. Perry, “Instantons and Seven-Branes in Type IIB Superstring Theory,” preprint hep-th/9511080.
- [18] J.X. Lu, Phys. Lett. **B313** (1993) 29.
- [19] F. Mandl and G. Shaw, *Quantum Field Theory* (John Wiley and Sons, 1984).
- [20] R.G. Leigh, Mod. Phys. Lett. **A4** (1989) 2767.
- [21] C. Schmidhuber, “D-brane actions,” preprint hep-th/9601003; A.A. Tseytlin, “Self-duality of Born-Infeld action and Dirichlet 3-brane of type IIB superstring theory,” preprint hep-th/9602064.
- [22] M. Veltman, in *Methods in field theory*, Les Houches 1975 Eds. R. Balian and J. Zinn-Justin (North Holland, 1976).
- [23] E. Bergshoeff, H.J. Boonstra and T. Ortín, “S-Duality and Dyonic p-Brane Solutions in Type II String Theory,” to appear in Phys. Rev. D, preprint hep-th/9508091.
- [24] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory* (Cambridge University Press, 1987).
- [25] C.G. Callan and I.R. Klebanov, “D-brane Boundary State Dynamics,” preprint hep-th/9511173.
- [26] M.B. Green, Phys. Lett. **B329** (1994) 435 [hep-th/9403040].